Developing confidence in critical state soil mechanics

7. CSSM done properly

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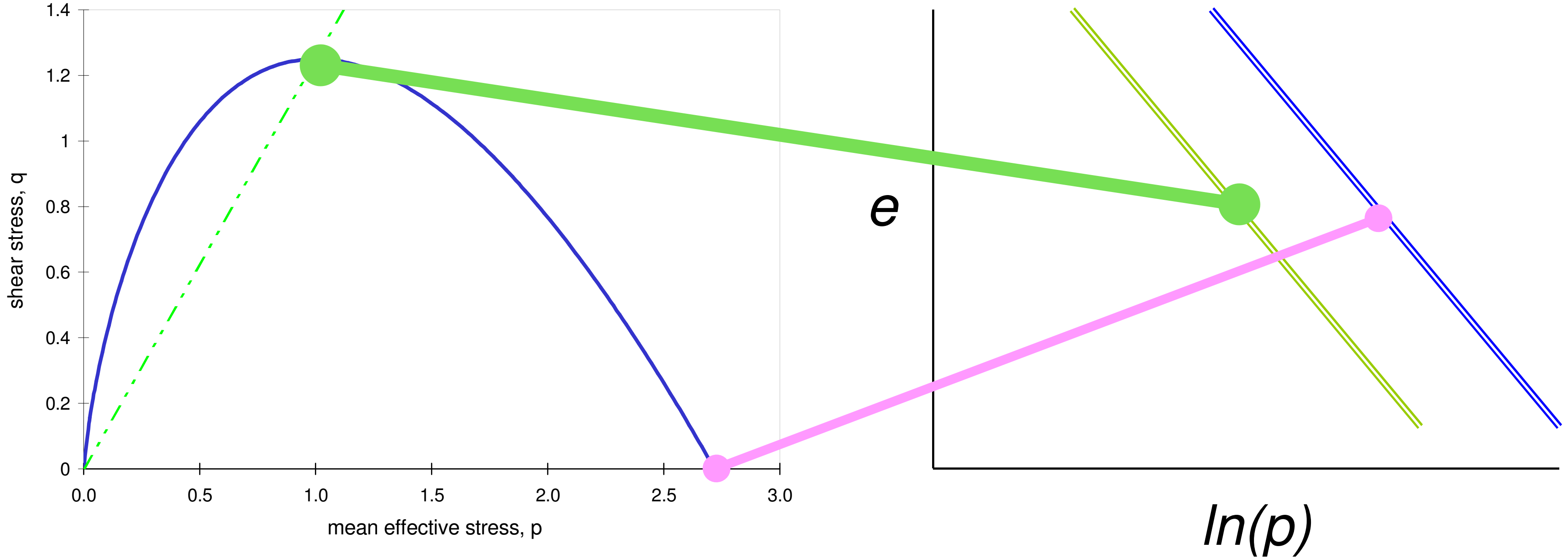
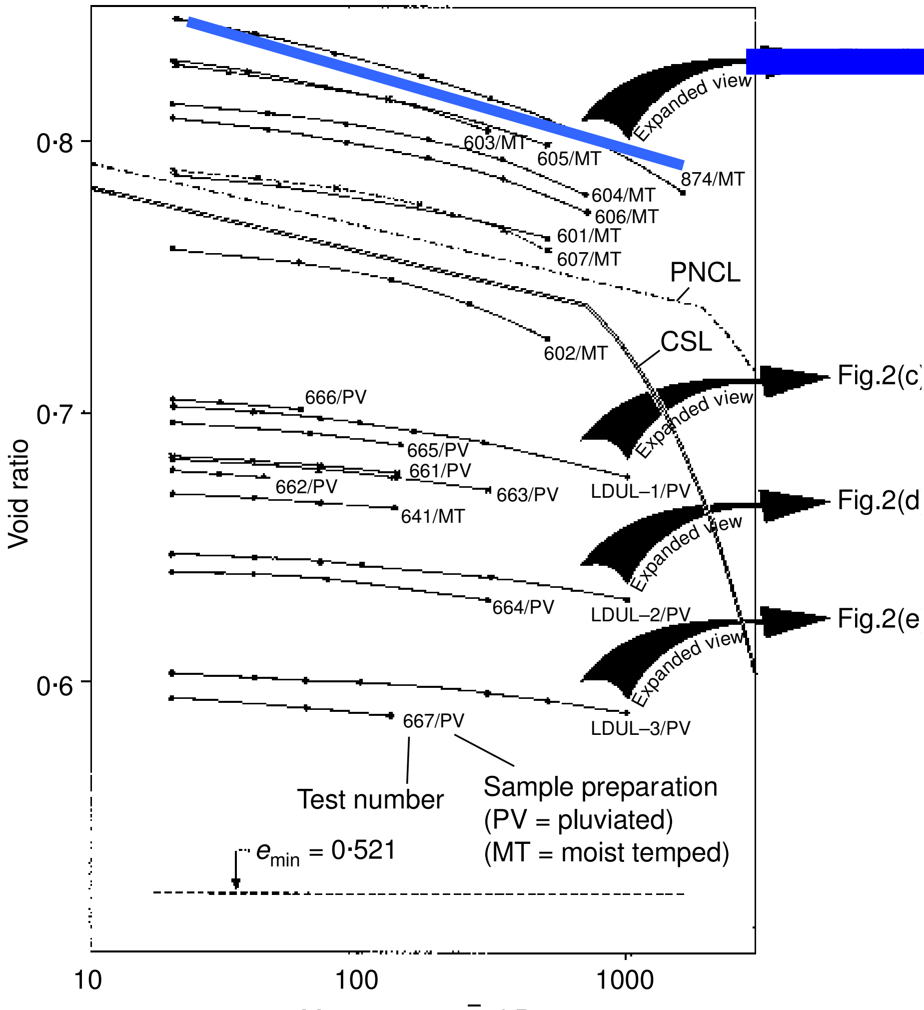
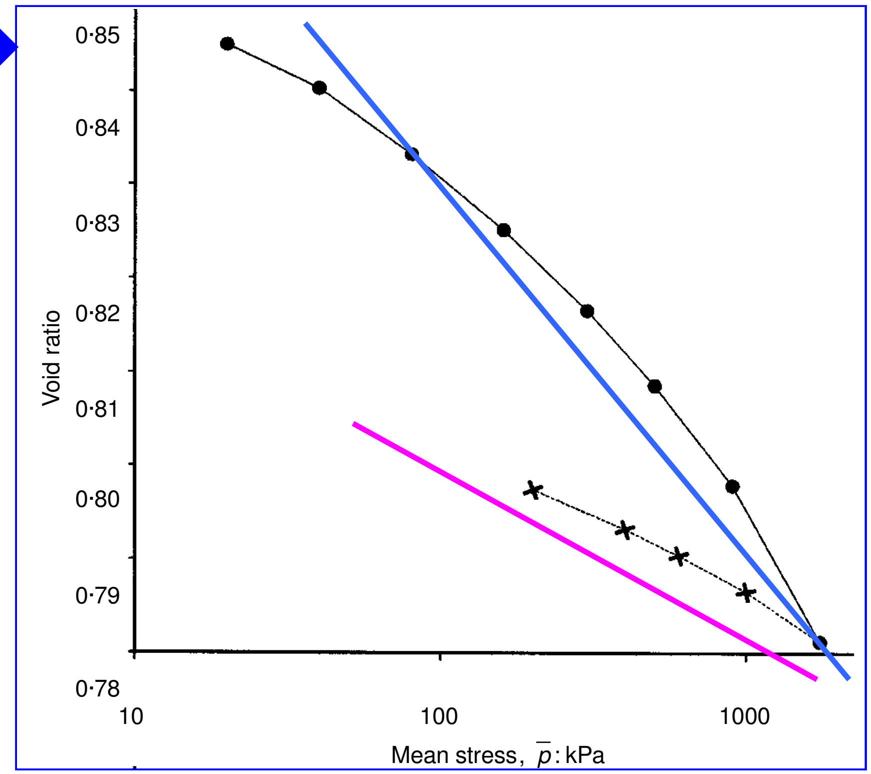
January, 2015

Basic premise of Cam Clay (OCC & MCC)

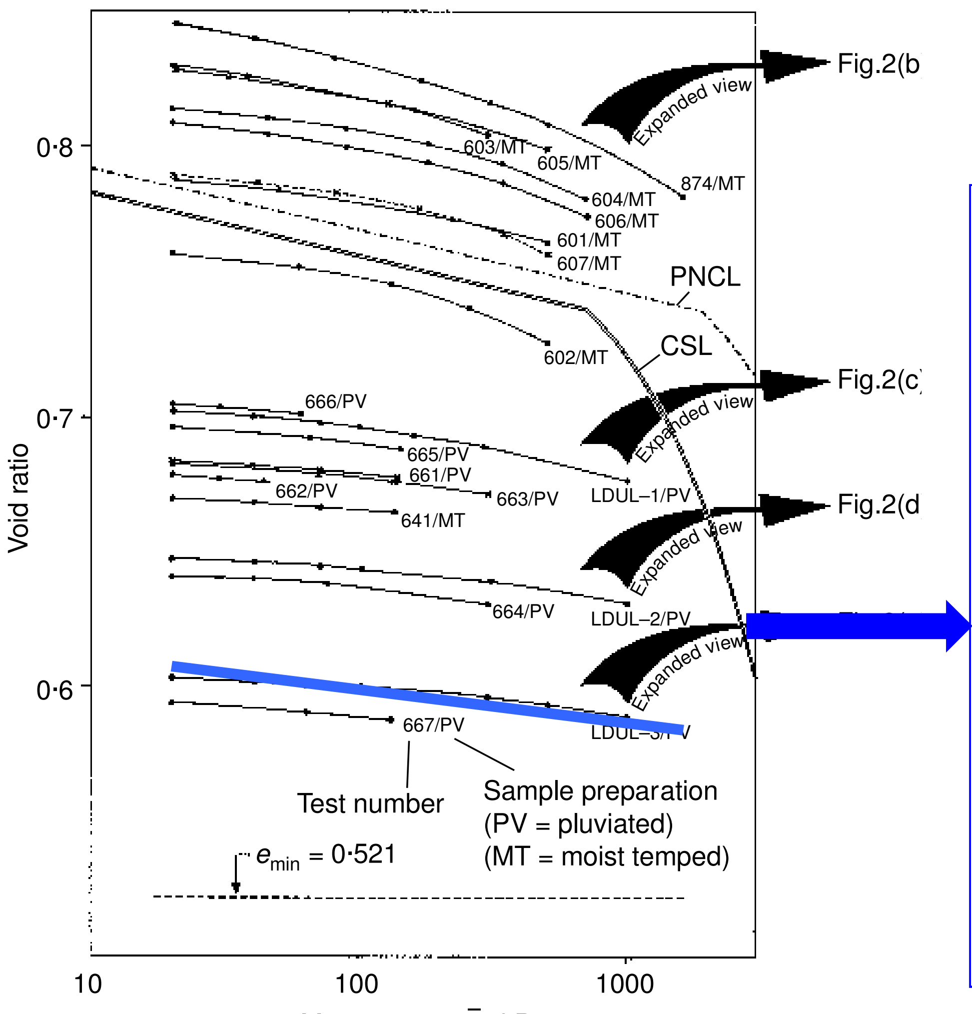


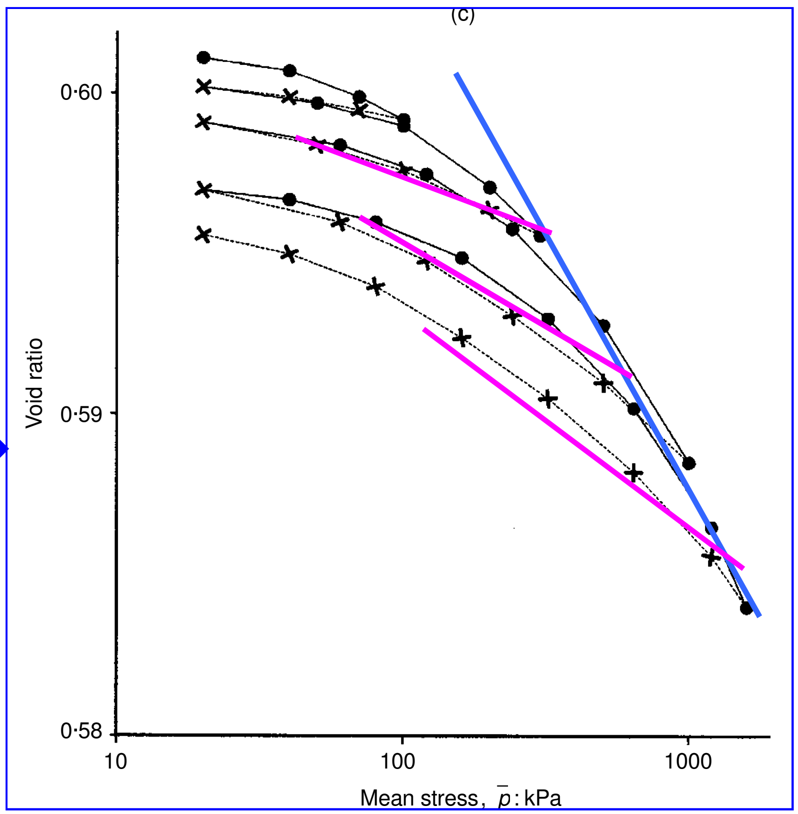
Soil behaviour in isotropic compression



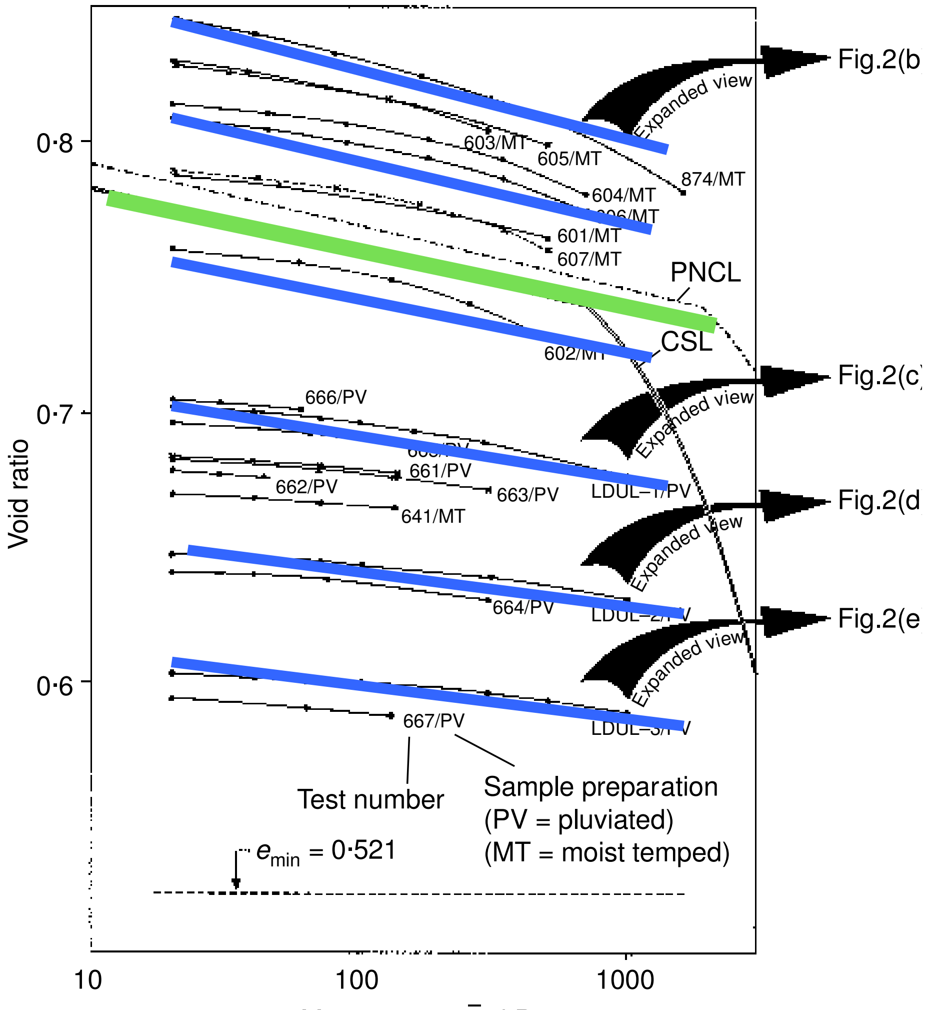


Soil behaviour in isotropic compression





Soil behaviour in isotropic compression



O'Tooles corollary to Murphy's Law

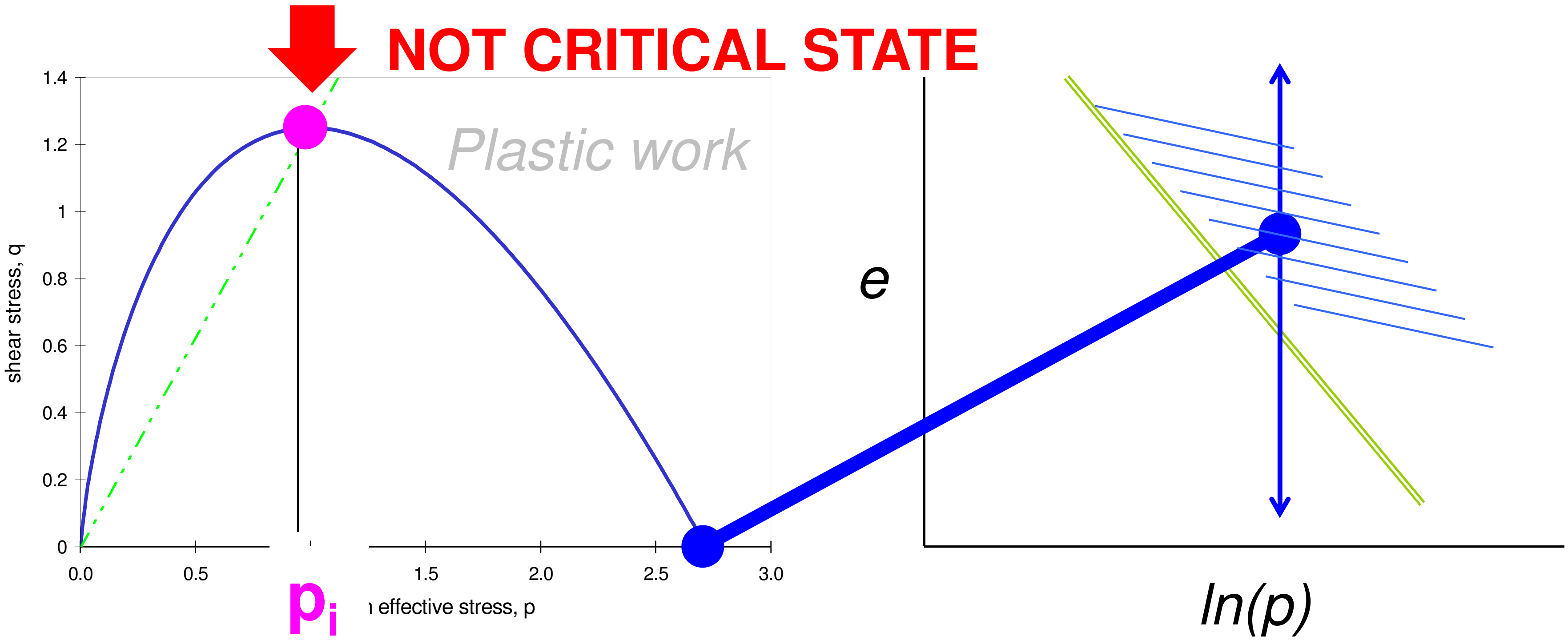
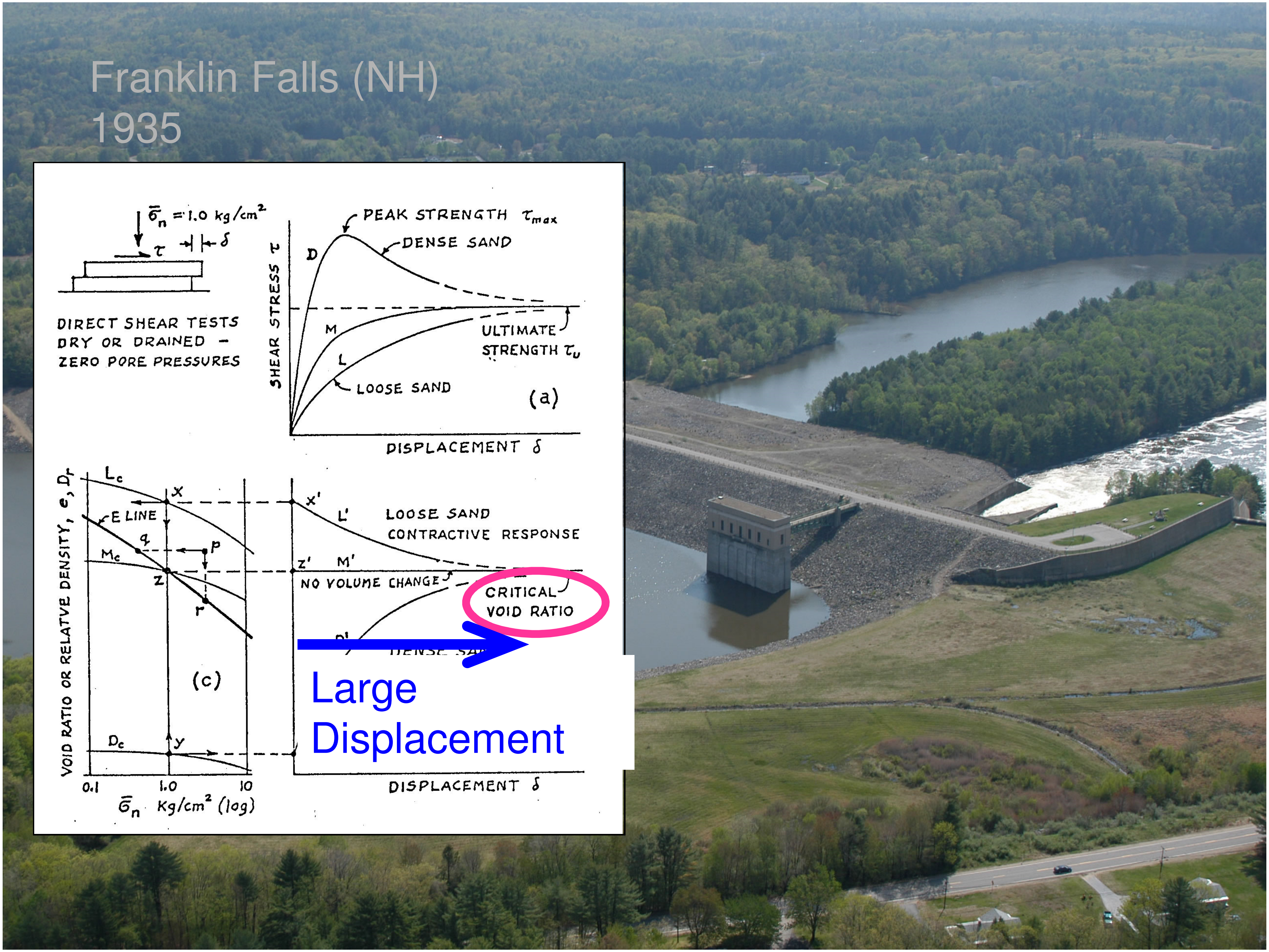


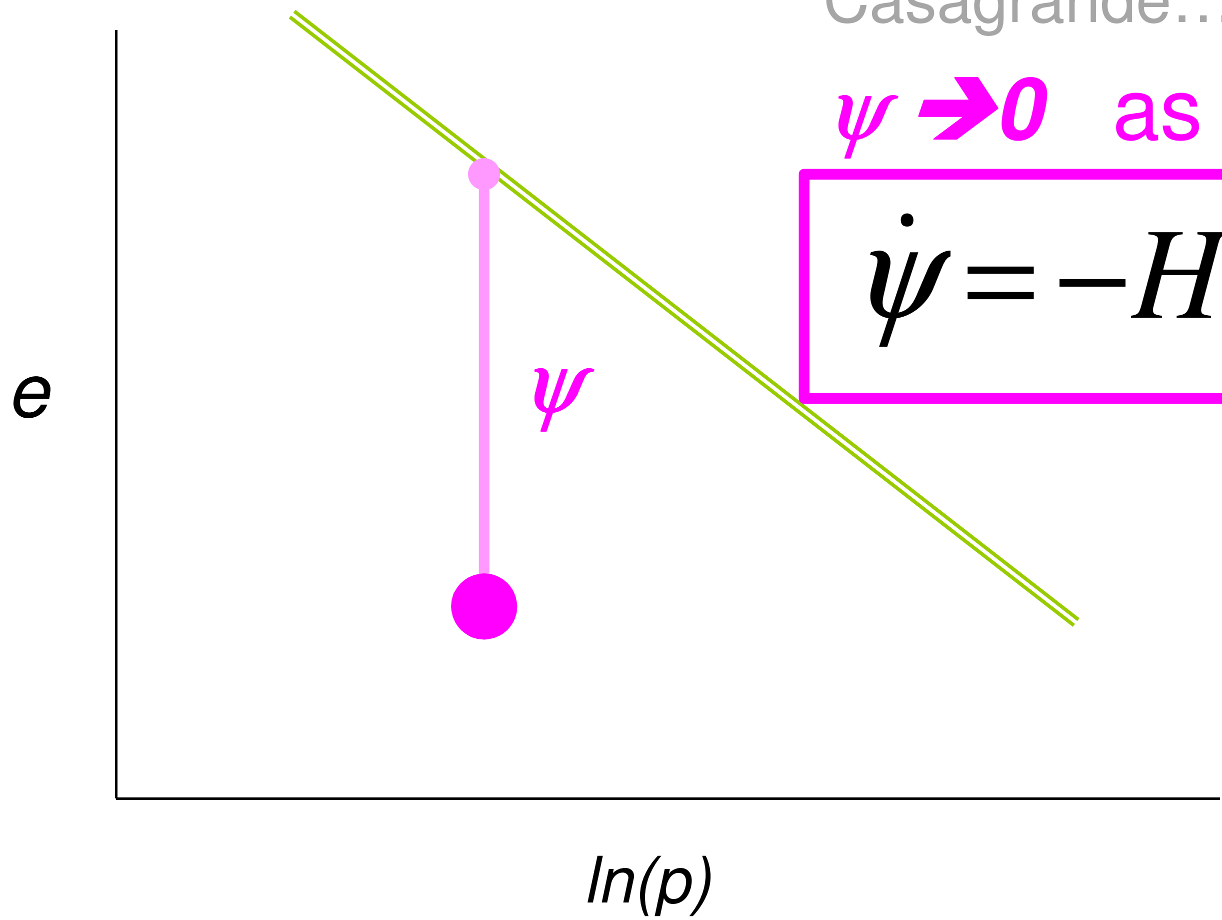
Image condition… $D^p = 0$

(violates $\Delta D^p/\Delta\varepsilon_q=0$)

aka 'pseudo steady state'

'phase change'

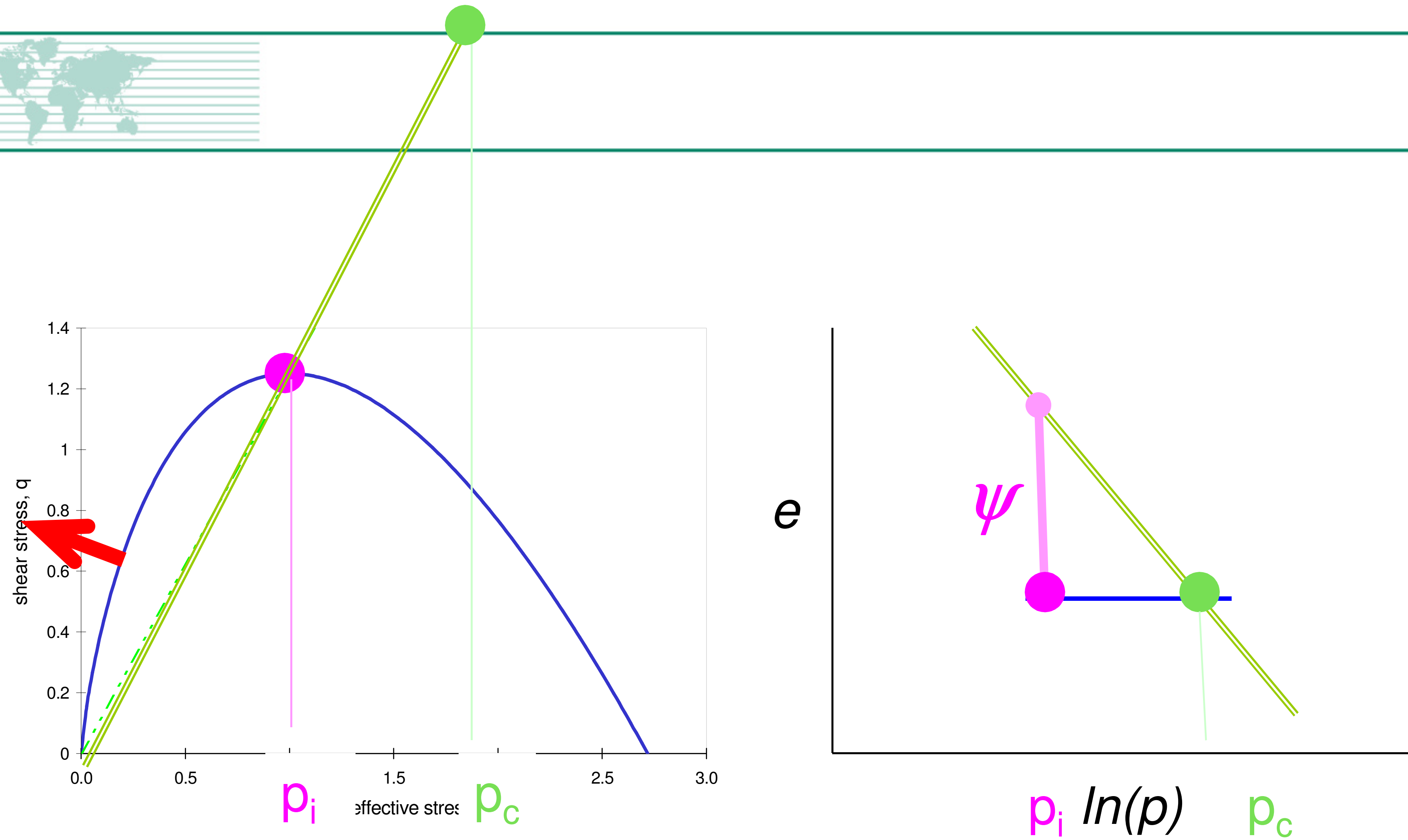

Franklin Falls (NH)
1935
$\bar{\sigma}_n = 1.0$ kg/cm²
τ
δ
DIRECT SHEAR TESTS
DRY OR DRAINED –
ZERO PORE PRESSURES
PEAK STRENGTH τ_{max}
DENSE SAND
SHEAR STRESS τ
D
M
L
ULTIMATE STRENGTH τ_u
LOOSE SAND
(a)
DISPLACEMENT δ
VOID RATIO OR RELATIVE DENSITY, e, D_r
L_c
X
E LINE
q
P
M_c
Z
r
(c)
D_c
y
0.1
1.0
10
$\bar{\sigma}_n$ Kg/cm² (log)
X'
L'
LOOSE SAND
CONTRACTIVE RESPONSE
Z'
M'
NO VOLUME CHANGE
CRITICAL VOID RATIO
Large Displacement
DISPLACEMENT δ



Casagrande…

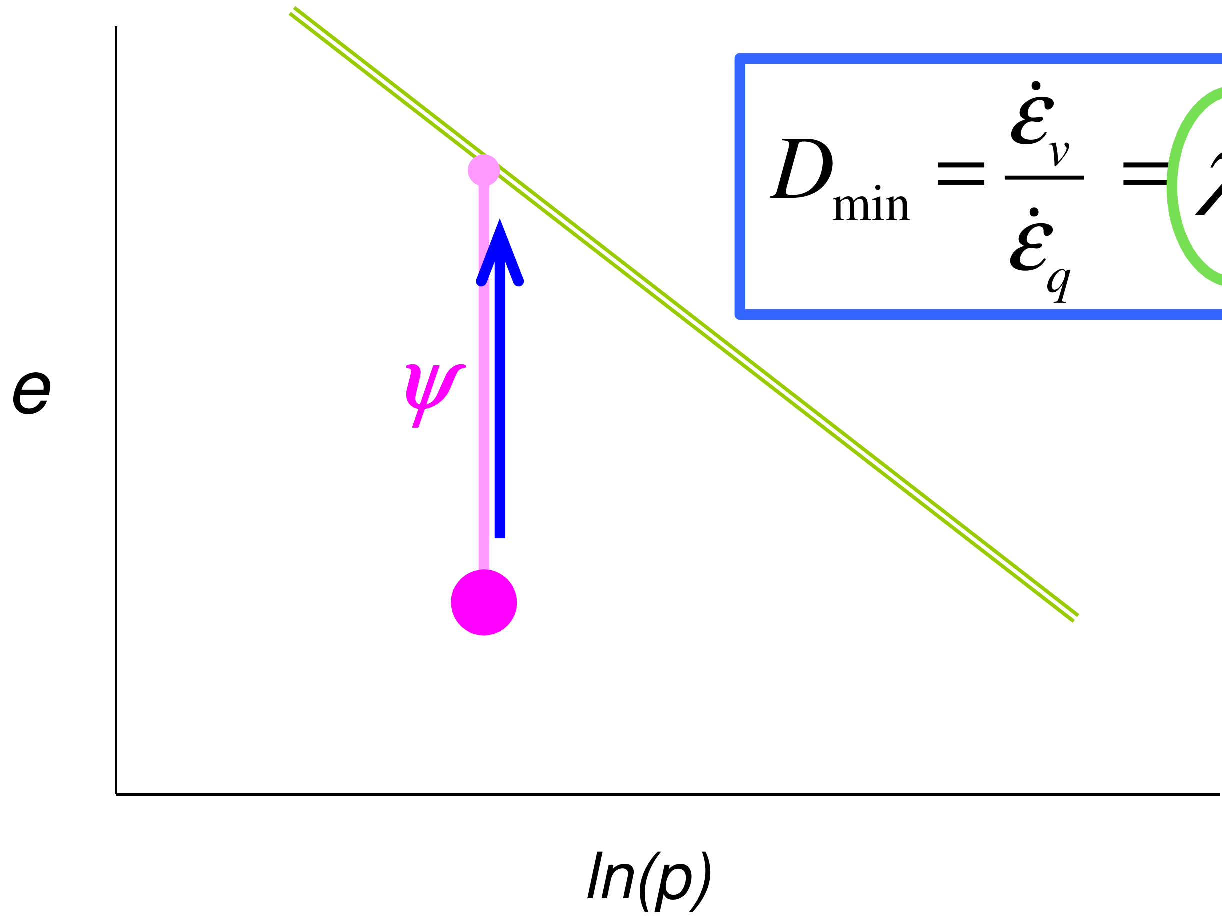
$\psi \rightarrow 0$ as $\varepsilon_\gamma \rightarrow$ □

$$\dot{\psi} = -H\,\psi\,\dot{\varepsilon}_q$$



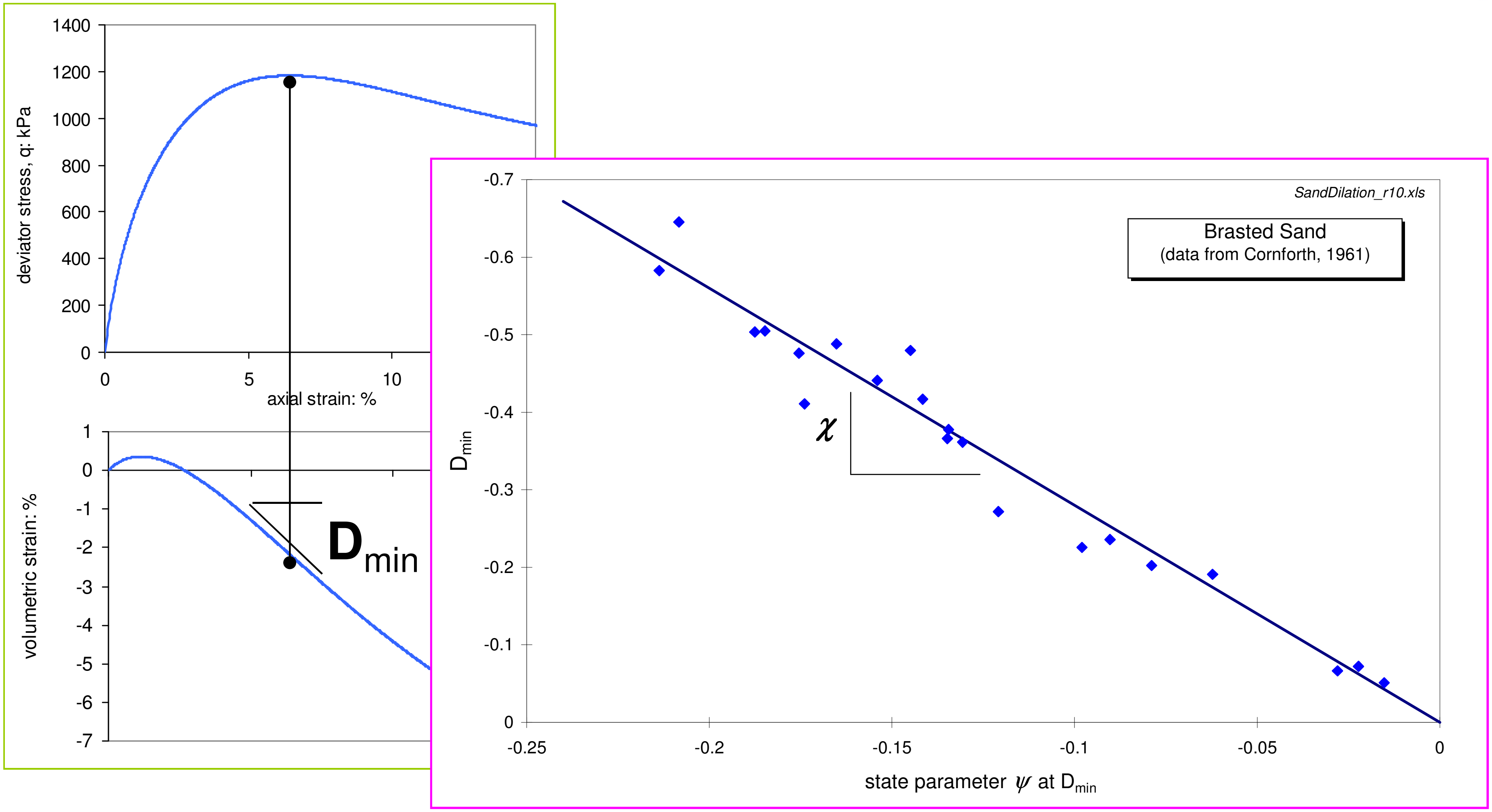
$$\dot{\psi} = -H\,\psi\,\dot{\varepsilon}_q \quad \Rightarrow \quad \dot{p}_i = -H\,p_i\,\ln(p_i/p_c)\,\dot{\varepsilon}_q$$

One more idea…

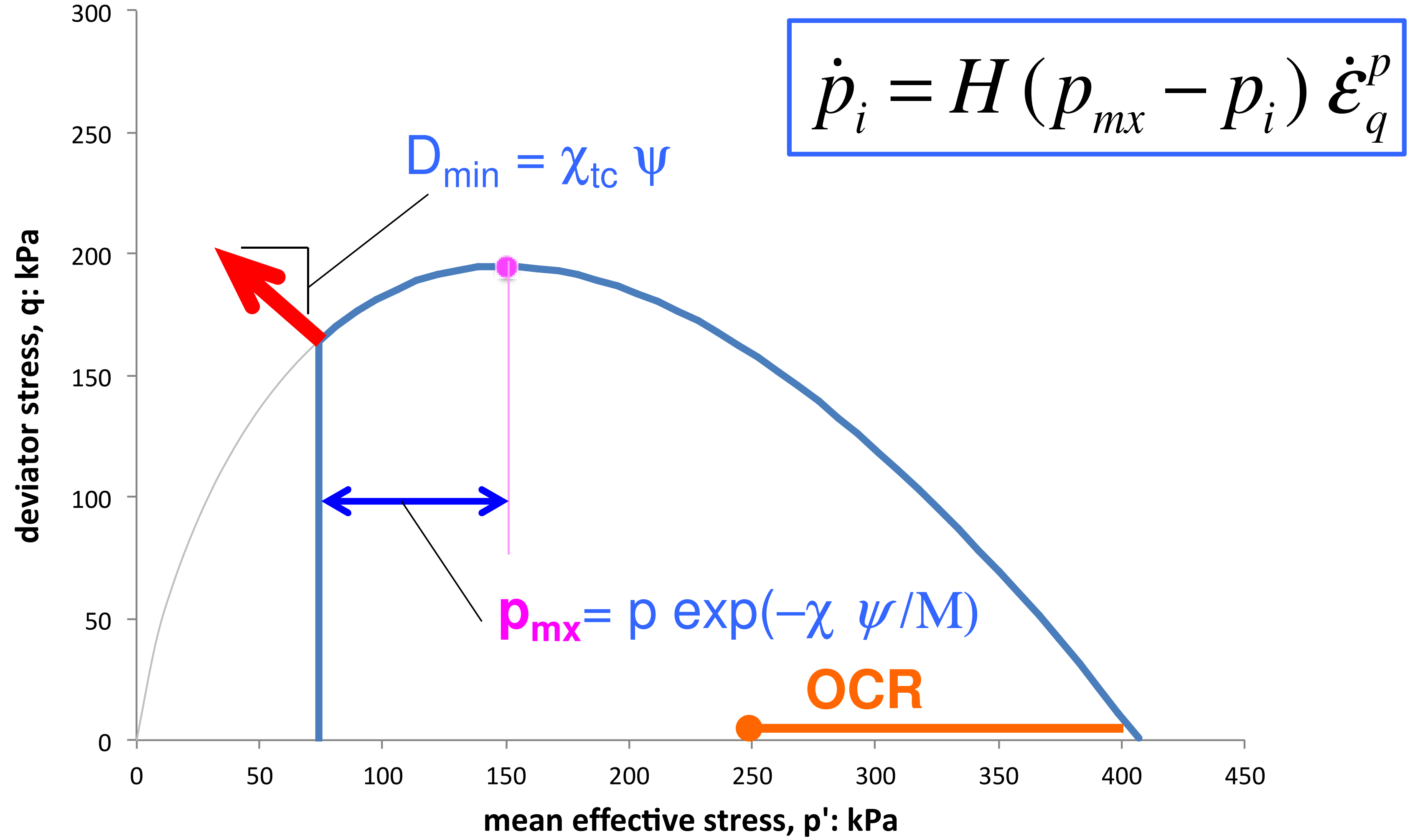


$$D_{\min} = \frac{\dot{\varepsilon}_v}{\dot{\varepsilon}_q} = \chi_{tc}\,\psi$$

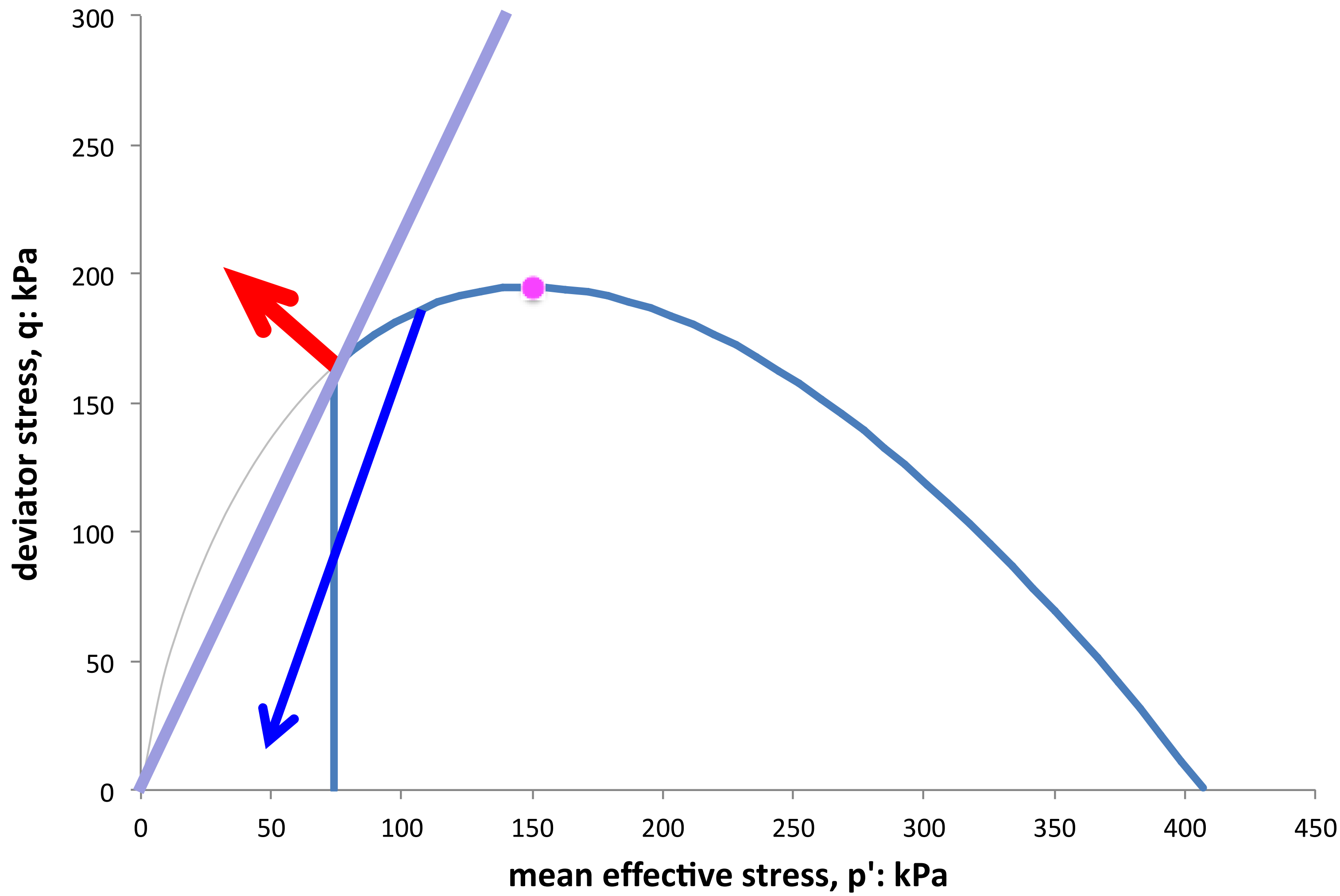
And the measured behaviour is…



NorSand (basic)



Hvorslev Surface



Original Cam Clay vs NorSand

	OCC	NS
Flowrule	$D^p = M - \eta$	$D^p = M - \eta$
Yield Surface	$\frac{\eta}{M} = 1 - \ln\left(\frac{p}{p_c}\right)$	$\frac{\eta}{M} = 1 - \ln\left(\frac{p}{p_i}\right)$
Hardening	$\dot{p}_c = \frac{1+e}{\lambda - \kappa} p_c D^p \dot{\varepsilon}_q^p$	$\dot{p}_i = H(p_{mx} - p_i)\,\dot{\varepsilon}_q^p$ $p_{mx} = p \exp(-\chi \psi / M)$
Elasticity	$K = \frac{1+e}{\kappa} p$; G = ∞	G = measured ; ν = constant $K = \frac{2G(1+\nu)}{3(1-2\nu)}$

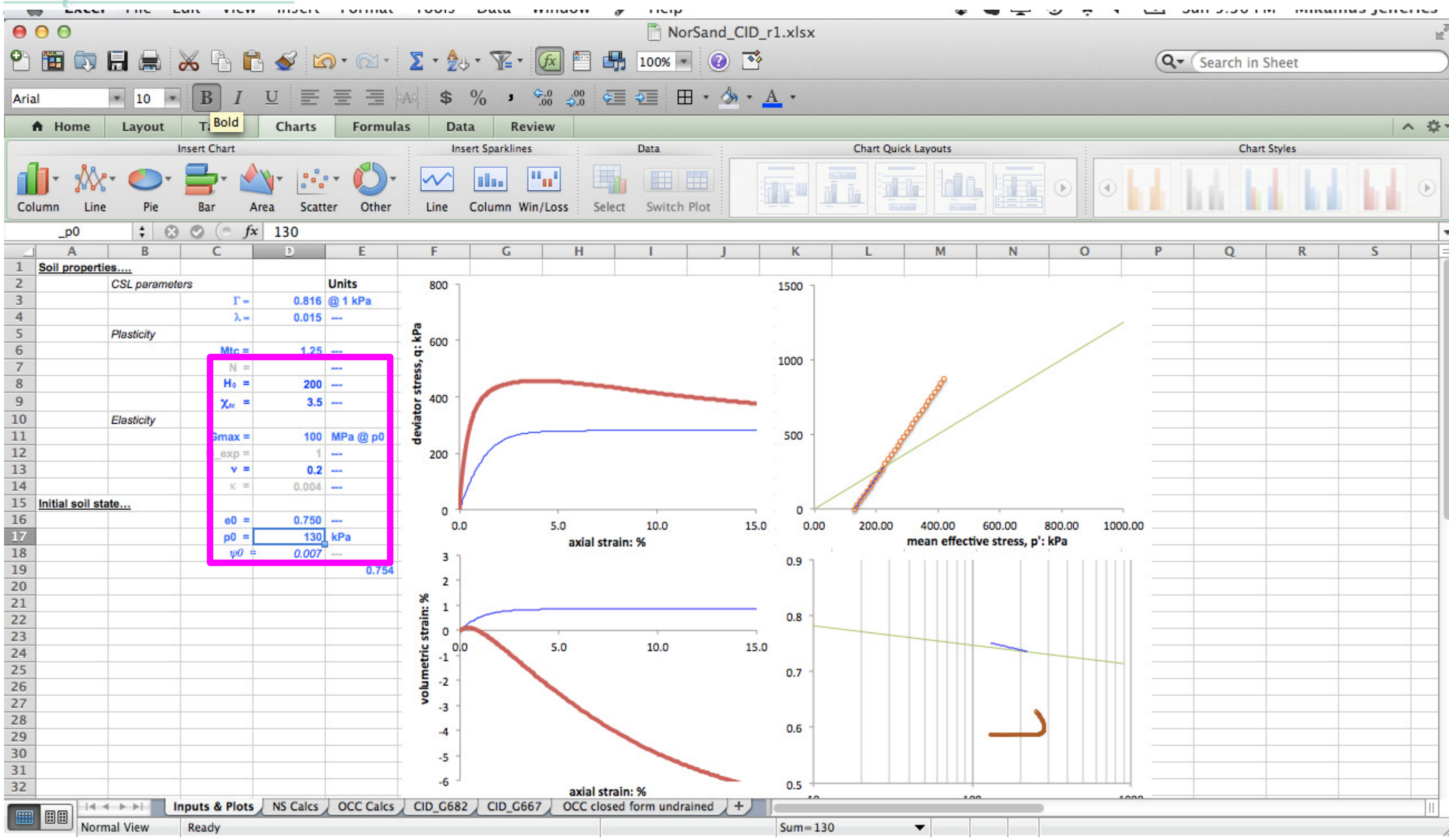
Over to you

- Make copy of drained OCC worksheet as 'NS Calcs'
  - Set graphs to address NS results

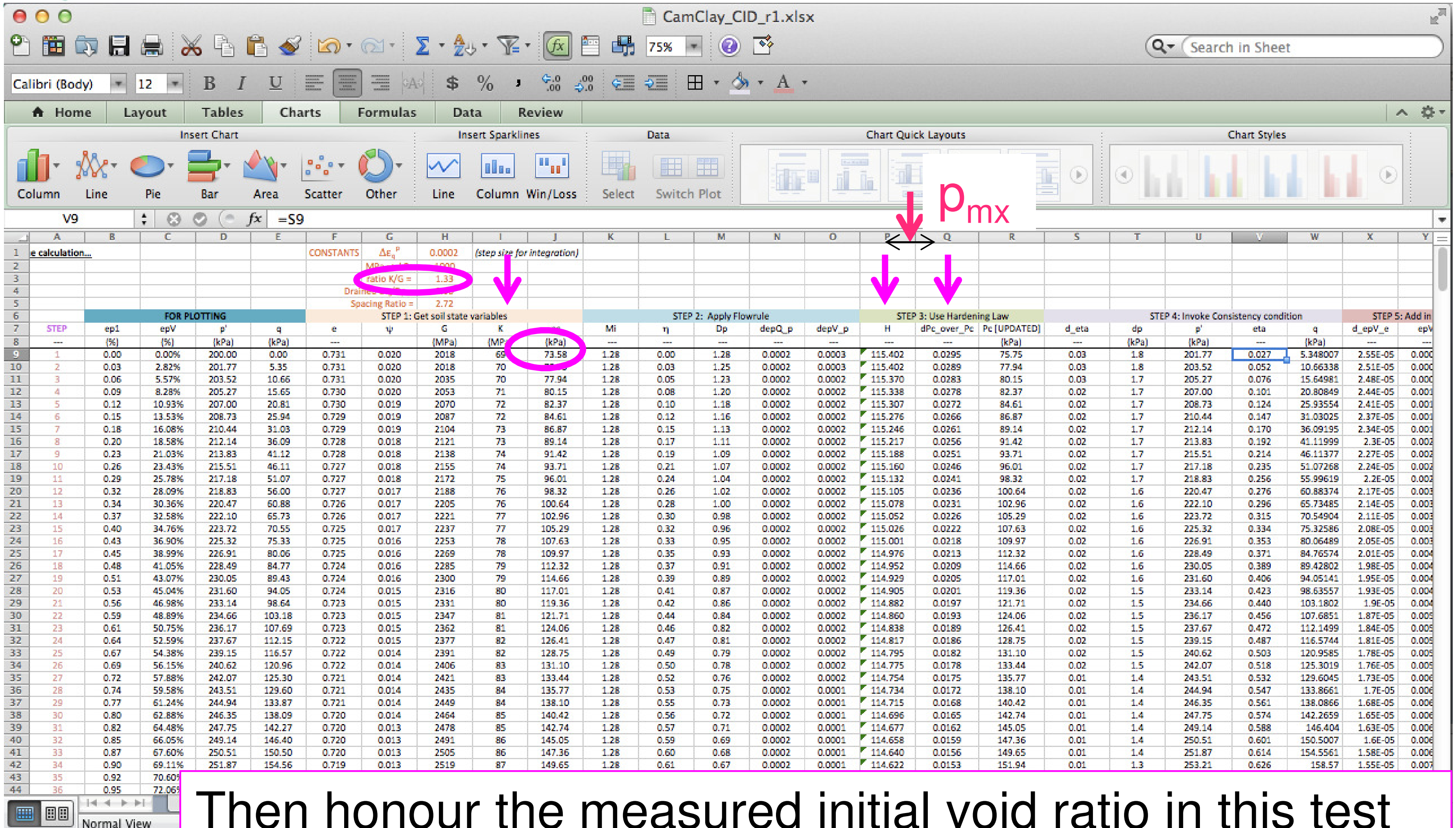
- Add in …
  - Soil properties H = 200, $\chi = 3.5$, G = 60 MPa, ν = 0.2
  - Column for p_{mx}
  - Calculation for ψ

- Modify
  - Column labels
  - Set: p_{img} initial = p_0 / SpacingRatio
  - Calc of p_i_dot / p_i (for new hardening law)
  - Calc of H (now just a constant)
  - Calc of K (see top of sheet)

Add soil properties



Drained OCC to NS… modifications

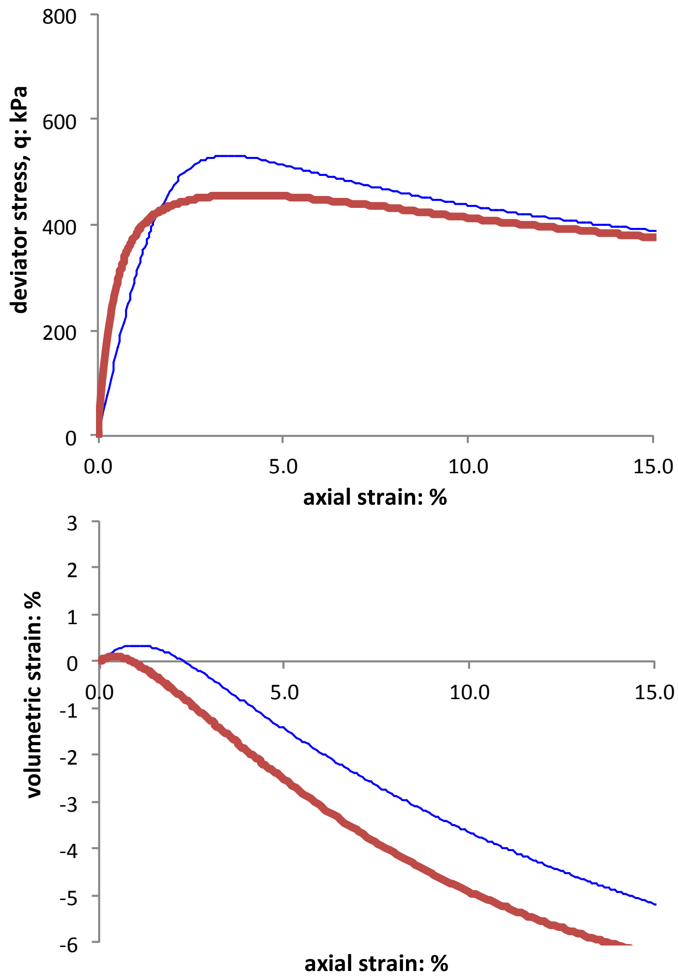


Row	A	B	C	D	E	F	G	H	I	J	K	L
1	e calculation...					CONSTANTS	$\Delta\varepsilon_q^p$	0.0002	(step size for integration)			
2							[illegible]	[illegible]				
3							ratio K/G =	1.33				
4							[illegible]	[illegible]				
5							Spacing Ratio =	2.72				
6		FOR PLOTTING				STEP 1: Get soil state variables					STEP 2: Apply Flowrule	
7	STEP	ep1	epV	p'	q	e	ψ	G	K		Mi	η
8	---	(%)	(%)	(kPa)	(kPa)	---		(MPa)	(MP	(kPa)	---	---
9	1	0.00	0.00%	200.00	0.00	0.731	0.020	2018	69	73.58	1.28	0.00
10	2	0.03	2.82%	201.77	5.35	0.731	0.020	2018	70	[illegible]	1.28	0.03
11	3	0.06	5.57%	203.52	10.66	0.731	0.020	2035	70	77.94	1.28	0.05
12	4	0.09	8.28%	205.27	15.65	0.730	0.020	2053	71	80.15	1.28	0.08
13	5	0.12	10.93%	207.00	20.81	0.730	0.019	2070	72	82.37	1.28	0.10
14	6	0.15	13.53%	208.73	25.94	0.729	0.019	2087	72	84.61	1.28	0.12
15	7	0.18	16.08%	210.44	31.03	0.729	0.019	2104	73	86.87	1.28	0.15
16	8	0.20	18.58%	212.14	36.09	0.728	0.018	2121	73	89.14	1.28	0.17
17	9	0.23	21.03%	213.83	41.12	0.728	0.018	2138	74	91.42	1.28	0.19
18	10	0.26	23.43%	215.51	46.11	0.727	0.018	2155	74	93.71	1.28	0.21
19	11	0.29	25.78%	217.18	51.07	0.727	0.018	2172	75	96.01	1.28	0.24
20	12	0.32	28.09%	218.83	56.00	0.727	0.017	2188	76	98.32	1.28	0.26
21	13	0.34	30.36%	220.47	60.88	0.726	0.017	2205	76	100.64	1.28	0.28
22	14	0.37	32.58%	222.10	65.73	0.726	0.017	2221	77	102.96	1.28	0.30
23	15	0.40	34.76%	223.72	70.55	0.725	0.017	2237	77	105.29	1.28	0.32
24	16	0.43	36.90%	225.32	75.33	0.725	0.016	2253	78	107.63	1.28	0.33
25	17	0.45	38.99%	226.91	80.06	0.725	0.016	2269	78	109.97	1.28	0.35
26	18	0.48	41.05%	228.49	84.77	0.724	0.016	2285	79	112.32	1.28	0.37
27	19	0.51	43.07%	230.05	89.43	0.724	0.016	2300	79	114.66	1.28	0.39
28	20	0.53	45.04%	231.60	94.05	0.724	0.015	2316	80	117.01	1.28	0.41
29	21	0.56	46.98%	233.14	98.64	0.723	0.015	2331	80	119.36	1.28	0.42
30	22	0.59	48.89%	234.66	103.18	0.723	0.015	2347	81	121.71	1.28	0.44
31	23	0.61	50.75%	236.17	107.69	0.723	0.015	2362	81	124.06	1.28	0.46
32	24	0.64	52.59%	237.67	112.15	0.722	0.015	2377	82	126.41	1.28	0.47
33	25	0.67	54.38%	239.15	116.57	0.722	0.014	2391	82	128.75	1.28	0.49
34	26	0.69	56.15%	240.62	120.96	0.722	0.014	2406	83	131.10	1.28	0.50
35	27	0.72	57.88%	242.07	125.30	0.721	0.014	2421	83	133.44	1.28	0.52
36	28	0.74	59.58%	243.51	129.60	0.721	0.014	2435	84	135.77	1.28	0.53
37	29	0.77	61.24%	244.94	133.87	0.721	0.014	2449	84	138.10	1.28	0.55
38	30	0.80	62.88%	246.35	138.09	0.720	0.014	2464	85	140.42	1.28	0.56
39	31	0.82	64.48%	247.75	142.27	0.720	0.013	2478	85	142.74	1.28	0.57
40	32	0.85	66.05%	249.14	146.40	0.720	0.013	2491	86	145.05	1.28	0.59
41	33	0.87	67.60%	250.51	150.50	0.720	0.013	2505	86	147.36	1.28	0.60
42	34	0.90	69.11%	251.87	154.56	0.719	0.013	2519	87	149.65	1.28	0.61
43	35	0.92	70.60									
44	36	0.95	72.06									

Row	M	N	O	P	Q	R	S	T	U	V	W	X	Y
6				STEP 3: Use Hardening Law			STEP 4: Invoke Consistency condition					STEP 5: Add in	
7	Dp	depQ_p	depV_p	H	dPc_over_Pc	Pc [UPDATED]	d_eta	dp	p'	eta	q	d_epV_e	epV
8	---	---	---	---	---	(kPa)	---	(kPa)	(kPa)	---	(kPa)	---	---
9	1.28	0.0002	0.0003	115.402	0.0295	75.75	0.03	1.8	201.77	0.027	5.348007	2.55E-05	0.000
10	1.25	0.0002	0.0003	115.402	0.0289	77.94	0.03	1.8	203.52	0.052	10.66338	2.51E-05	0.000
11	1.23	0.0002	0.0002	115.370	0.0283	80.15	0.03	1.7	205.27	0.076	15.64981	2.48E-05	0.000
12	1.20	0.0002	0.0002	115.338	0.0278	82.37	0.02	1.7	207.00	0.101	20.80849	2.44E-05	0.001
13	1.18	0.0002	0.0002	115.307	0.0272	84.61	0.02	1.7	208.73	0.124	25.93554	2.41E-05	0.001
14	1.16	0.0002	0.0002	115.276	0.0266	86.87	0.02	1.7	210.44	0.147	31.03025	2.37E-05	0.001
15	1.13	0.0002	0.0002	115.246	0.0261	89.14	0.02	1.7	212.14	0.170	36.09195	2.34E-05	0.001
16	1.11	0.0002	0.0002	115.217	0.0256	91.42	0.02	1.7	213.83	0.192	41.11999	2.3E-05	0.002
17	1.09	0.0002	0.0002	115.188	0.0251	93.71	0.02	1.7	215.51	0.214	46.11377	2.27E-05	0.002
18	1.07	0.0002	0.0002	115.160	0.0246	96.01	0.02	1.7	217.18	0.235	51.07268	2.24E-05	0.002
19	1.04	0.0002	0.0002	115.132	0.0241	98.32	0.02	1.7	218.83	0.256	55.99619	2.2E-05	0.002
20	1.02	0.0002	0.0002	115.105	0.0236	100.64	0.02	1.6	220.47	0.276	60.88374	2.17E-05	0.003
21	1.00	0.0002	0.0002	115.078	0.0231	102.96	0.02	1.6	222.10	0.296	65.73485	2.14E-05	0.003
22	0.98	0.0002	0.0002	115.052	0.0226	105.29	0.02	1.6	223.72	0.315	70.54904	2.11E-05	0.003
23	0.96	0.0002	0.0002	115.026	0.0222	107.63	0.02	1.6	225.32	0.334	75.32586	2.08E-05	0.003
24	0.95	0.0002	0.0002	115.001	0.0218	109.97	0.02	1.6	226.91	0.353	80.06489	2.05E-05	0.003
25	0.93	0.0002	0.0002	114.976	0.0213	112.32	0.02	1.6	228.49	0.371	84.76574	2.01E-05	0.004
26	0.91	0.0002	0.0002	114.952	0.0209	114.66	0.02	1.6	230.05	0.389	89.42802	1.98E-05	0.004
27	0.89	0.0002	0.0002	114.929	0.0205	117.01	0.02	1.6	231.60	0.406	94.05141	1.95E-05	0.004
28	0.87	0.0002	0.0002	114.905	0.0201	119.36	0.02	1.5	233.14	0.423	98.63557	1.93E-05	0.004
29	0.86	0.0002	0.0002	114.882	0.0197	121.71	0.02	1.5	234.66	0.440	103.1802	1.9E-05	0.004
30	0.84	0.0002	0.0002	114.860	0.0193	124.06	0.02	1.5	236.17	0.456	107.6851	1.87E-05	0.005
31	0.82	0.0002	0.0002	114.838	0.0189	126.41	0.02	1.5	237.67	0.472	112.1499	1.84E-05	0.005
32	0.81	0.0002	0.0002	114.817	0.0186	128.75	0.02	1.5	239.15	0.487	116.5744	1.81E-05	0.005
33	0.79	0.0002	0.0002	114.795	0.0182	131.10	0.02	1.5	240.62	0.503	120.9585	1.78E-05	0.005
34	0.78	0.0002	0.0002	114.775	0.0178	133.44	0.02	1.5	242.07	0.518	125.3019	1.76E-05	0.005
35	0.76	0.0002	0.0002	114.754	0.0175	135.77	0.01	1.4	243.51	0.532	129.6045	1.73E-05	0.006
36	0.75	0.0002	0.0001	114.734	0.0172	138.10	0.01	1.4	244.94	0.547	133.8661	1.7E-05	0.006
37	0.73	0.0002	0.0001	114.715	0.0168	140.42	0.01	1.4	246.35	0.561	138.0866	1.68E-05	0.006
38	0.72	0.0002	0.0001	114.696	0.0165	142.74	0.01	1.4	247.75	0.574	142.2659	1.65E-05	0.006
39	0.71	0.0002	0.0001	114.677	0.0162	145.05	0.01	1.4	249.14	0.588	146.404	1.63E-05	0.006
40	0.69	0.0002	0.0001	114.658	0.0159	147.36	0.01	1.4	250.51	0.601	150.5007	1.6E-05	0.006
41	0.68	0.0002	0.0001	114.640	0.0156	149.65	0.01	1.4	251.87	0.614	154.5561	1.58E-05	0.006
42	0.67	0.0002	0.0001	114.622	0.0153	151.94	0.01	1.3	253.21	0.626	158.57	1.55E-05	0.007

Then honour the measured initial void ratio in this test CID_667 (by freezing method)… **SET: e_0 = 0.587**

Computed soil response for simple NS



Let’s get NorSand working

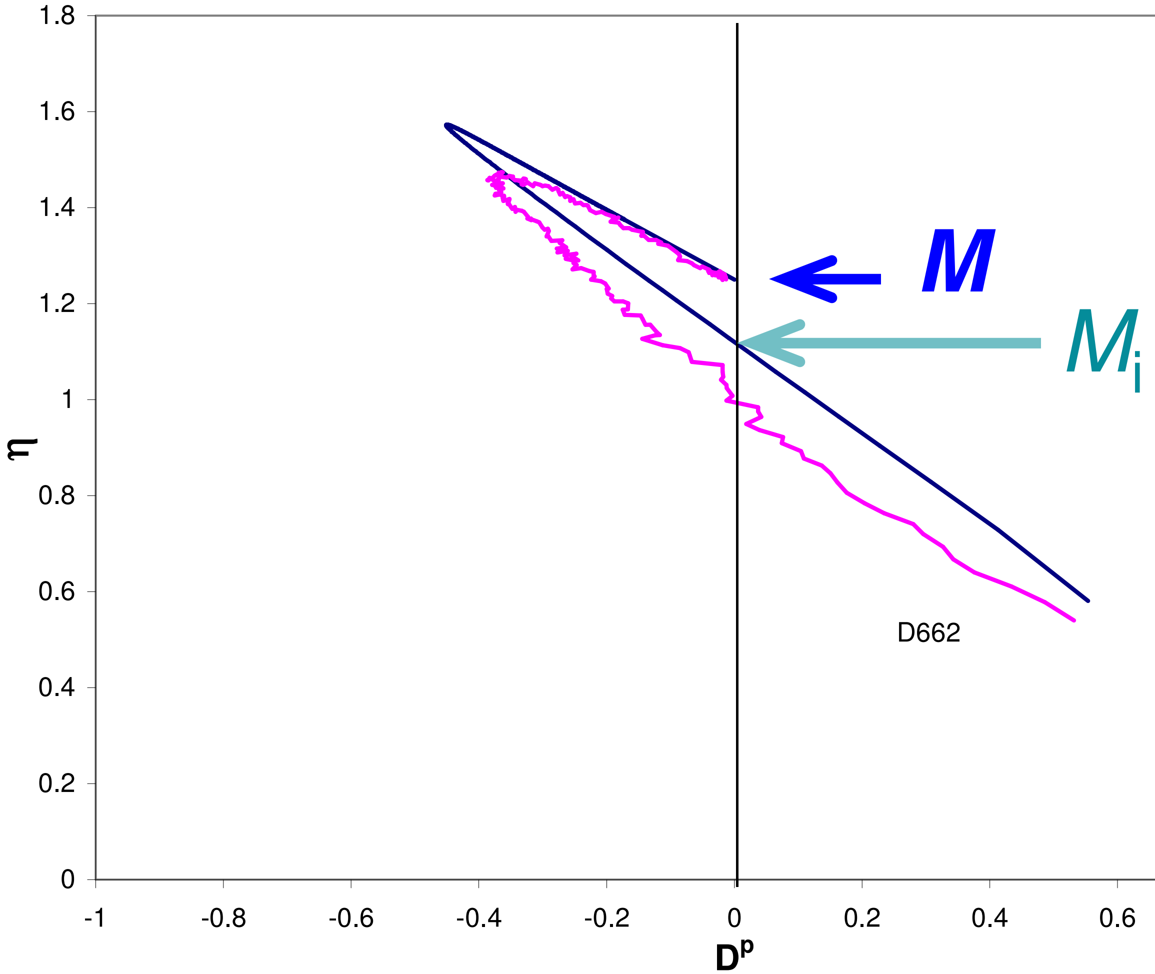
(and then we will tweak)

Tweaks

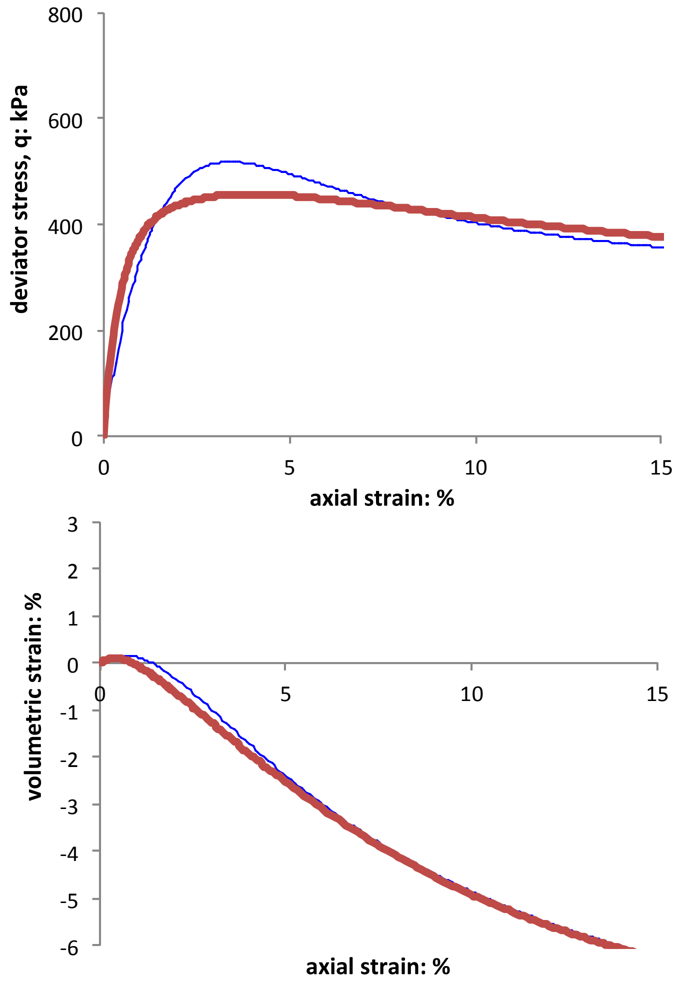
- There is an element of shear hardening when matching pure theory to data
  - FIX: $H = H\, p / p_{img}$

- The coefficient ‘M’ (or ϕ_c) is not a constant in stress-dilatancy theory
  - Known since ~1963
  - Dafalias 1997 suggests $M(\psi)$
  - Nova’s flowrule best for peak strength… *N*
  - FIX: $M_i = M - N \chi \psi$

Stress dilatancy



Fit to test


800
600
400
200
0
deviator stress, q: kPa
0
5
10
15
axial strain: %
3
2
1
0
-1
-2
-3
-4
-5
-6
volumetric strain: %
0
5
10
15
axial strain: %

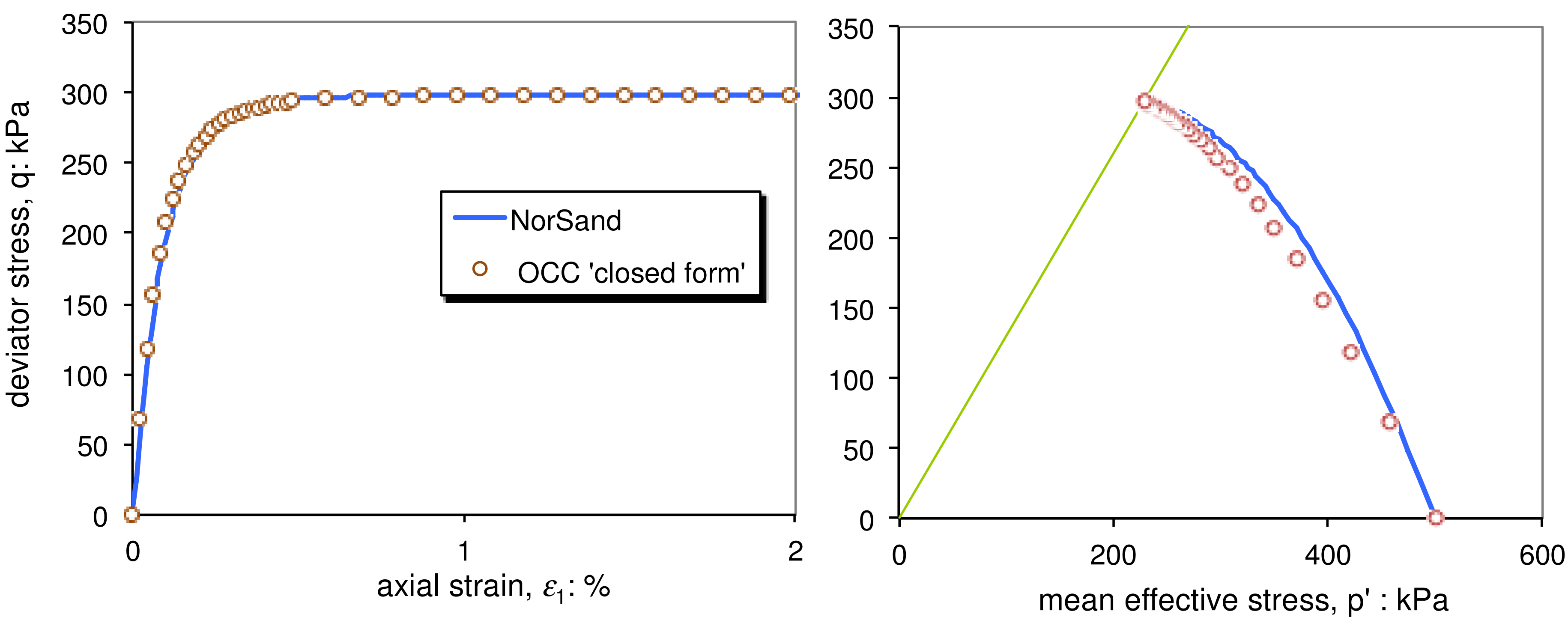
"Horses for courses"

- "Cam Clay is crap"
    - OCC "fine for soft clays"
    - NS "best in class" for sands

- CSSM
    - Based on axioms and derivations
    - No specification or role for soil gradation (or fines…)
    - "clay" is just a different set of soil properties from "sand"

- OCC is a specific (degenerate) case of NS
    - $N = 0$
    - $\psi_0 = \lambda - \kappa$
    - $H = 1/(\lambda - \kappa)$
    - G = large number & ν set to small number to recover κ

OCC as special case of NS



See J&B Append H in workshop notes