# Developing confidence in critical state soil mechanics

## 6. Drained txl compression of OCC ‘*consistency condition*’

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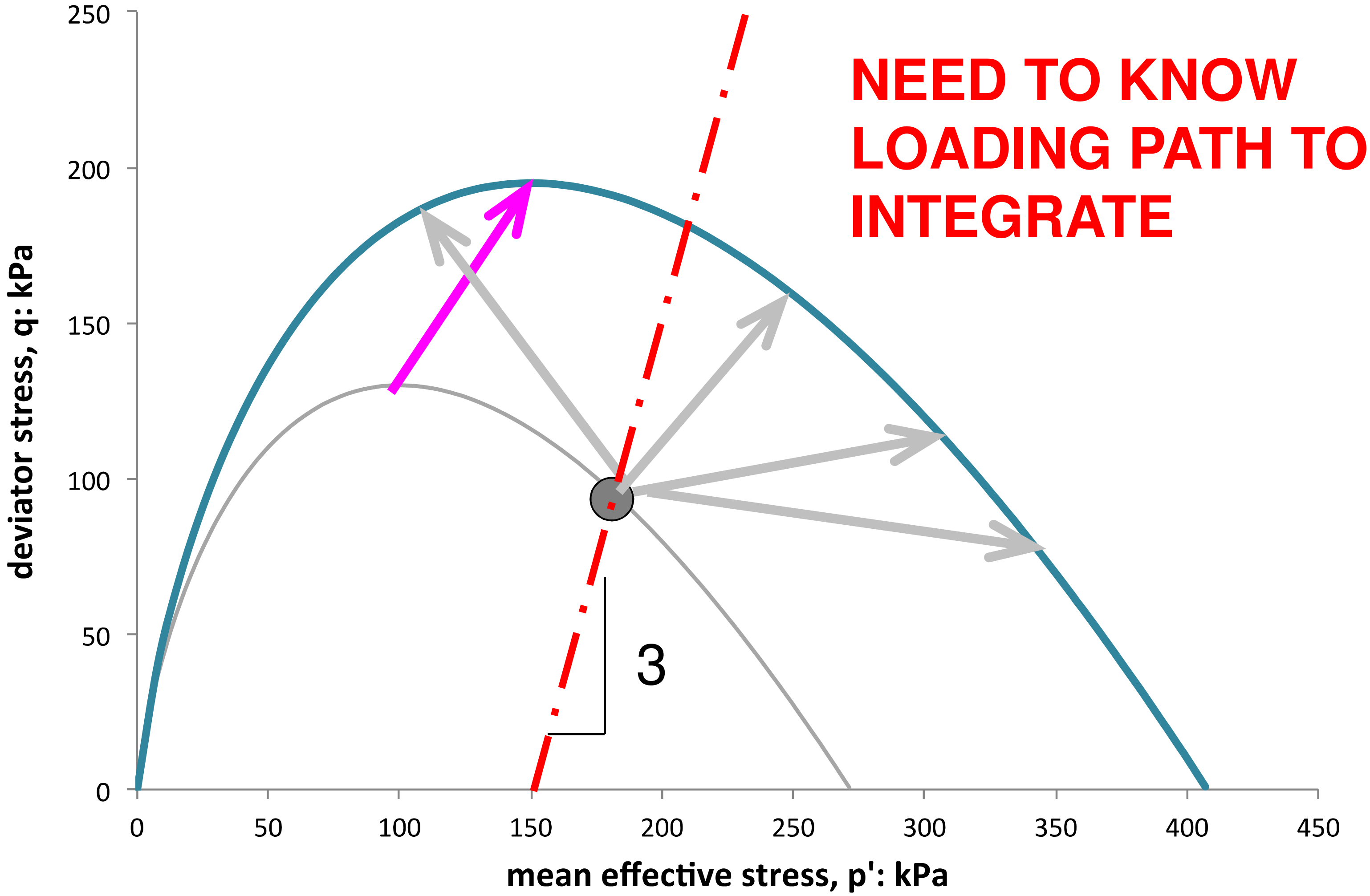
# Consistency condition



# Key ideas (equations…)

- Yield surface: F(σ, *e*, ζ)
  - If F < 0 then 'elastic'
  - If F=0 then 'plastic'
  - F>0 not admissible

- CONSISTENCY CONDITION
  - Must remain on the yield surface during yielding
  - mathematically: dF = 0

- Rewrite OCC yield surface as:

$$F = 1 - \frac{\eta}{M} - \ln\left(\frac{p}{p_c}\right)$$

# Consistency condition for OCC

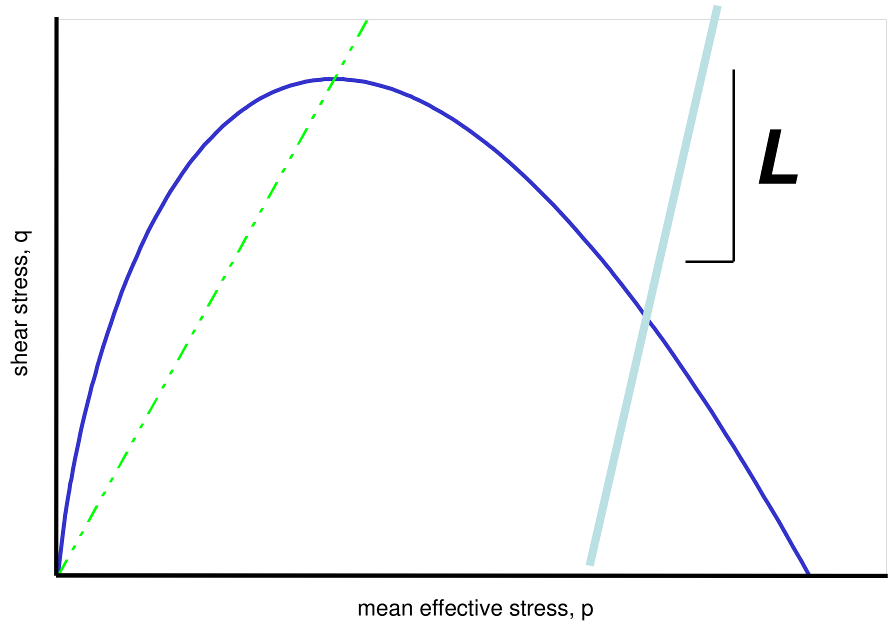
*For yield surface written as F=0,* ***consistency is dF=0***

$$F = 1 - \frac{\eta}{M} - \ln\left(\frac{p}{p_c}\right) \quad \Rightarrow \quad dF = -\frac{d\eta}{M} - \left(\frac{p}{p_c}\right)^{-1} d\left(\frac{p}{p_c}\right)$$

$$\Rightarrow \quad \boxed{\dot{\eta} = M\left(\frac{\dot{p}_c}{p_c} - \frac{\dot{p}}{p}\right)}$$

*Use with...* $\eta_{j+1} = \eta_j + \dot{\eta}_j$

# Consistency in drained triaxial



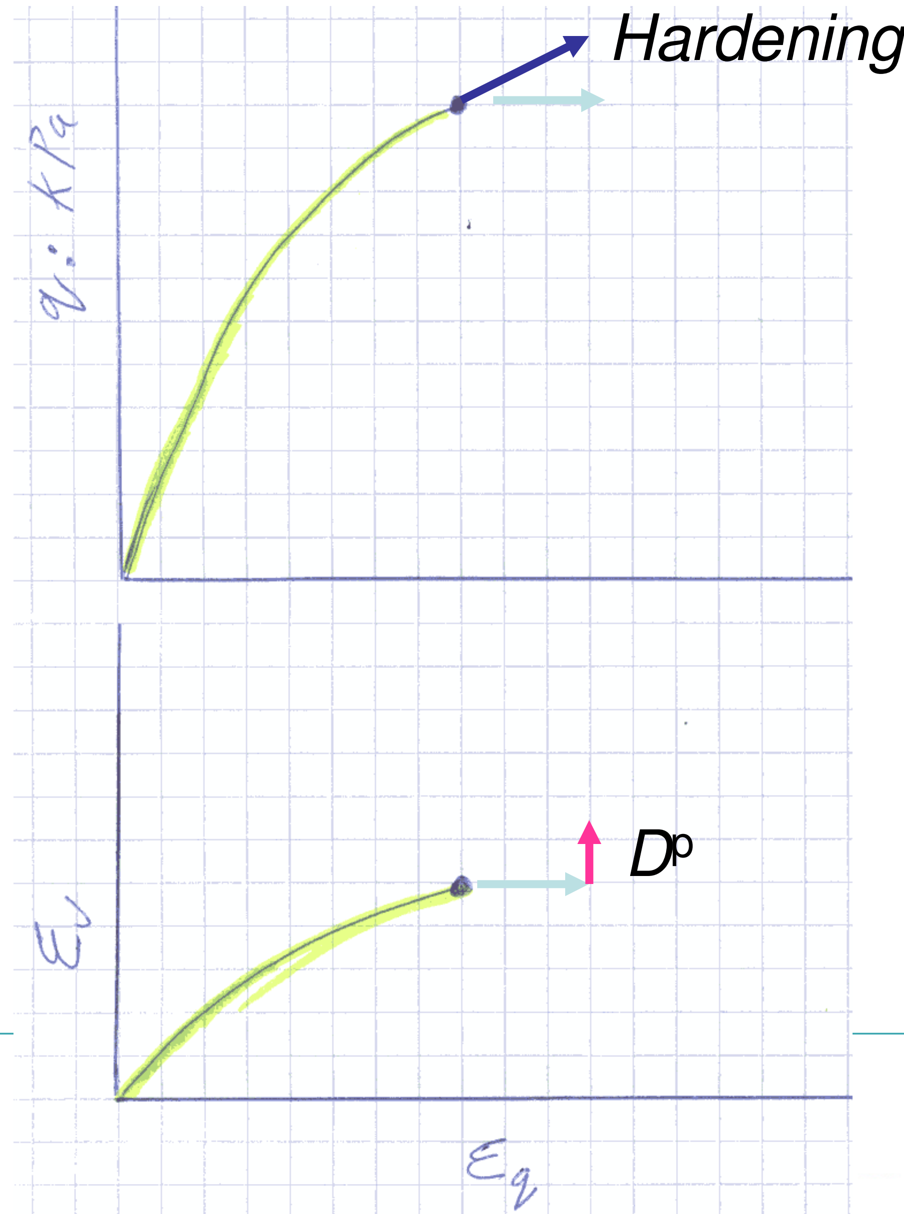

*Differentiate* $\eta = q/p$

$$\Rightarrow \qquad \frac{\dot{p}}{p} = \frac{\dot{\eta}}{L-\eta}$$

Substitute in OCC consistency condition

$$\dot{\eta}\left(\frac{1}{M} + \frac{1}{L-\eta}\right) = \frac{\dot{p}_c}{p_c}$$

# Euler method for OCC (forward difference)

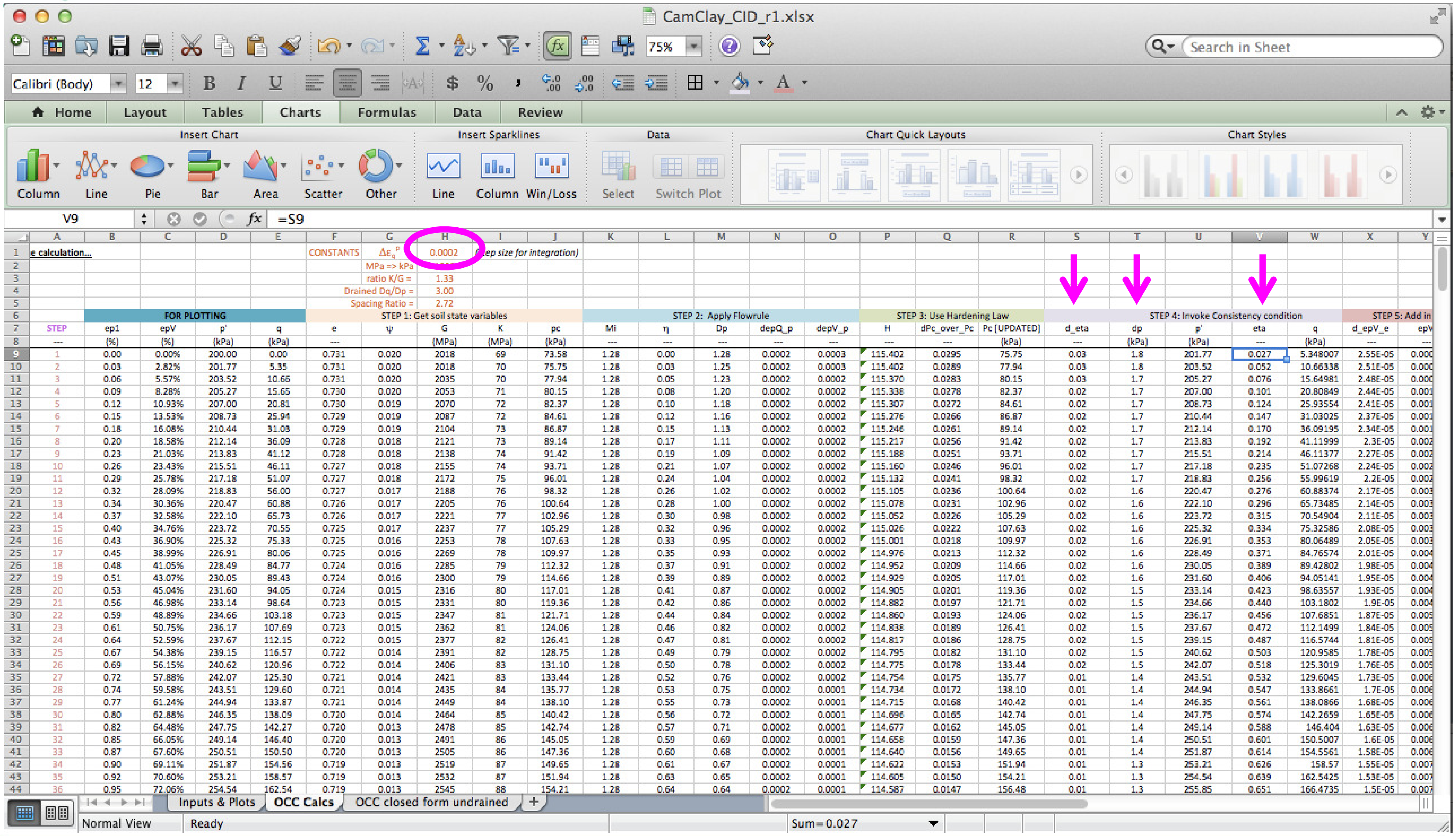


# Over to you

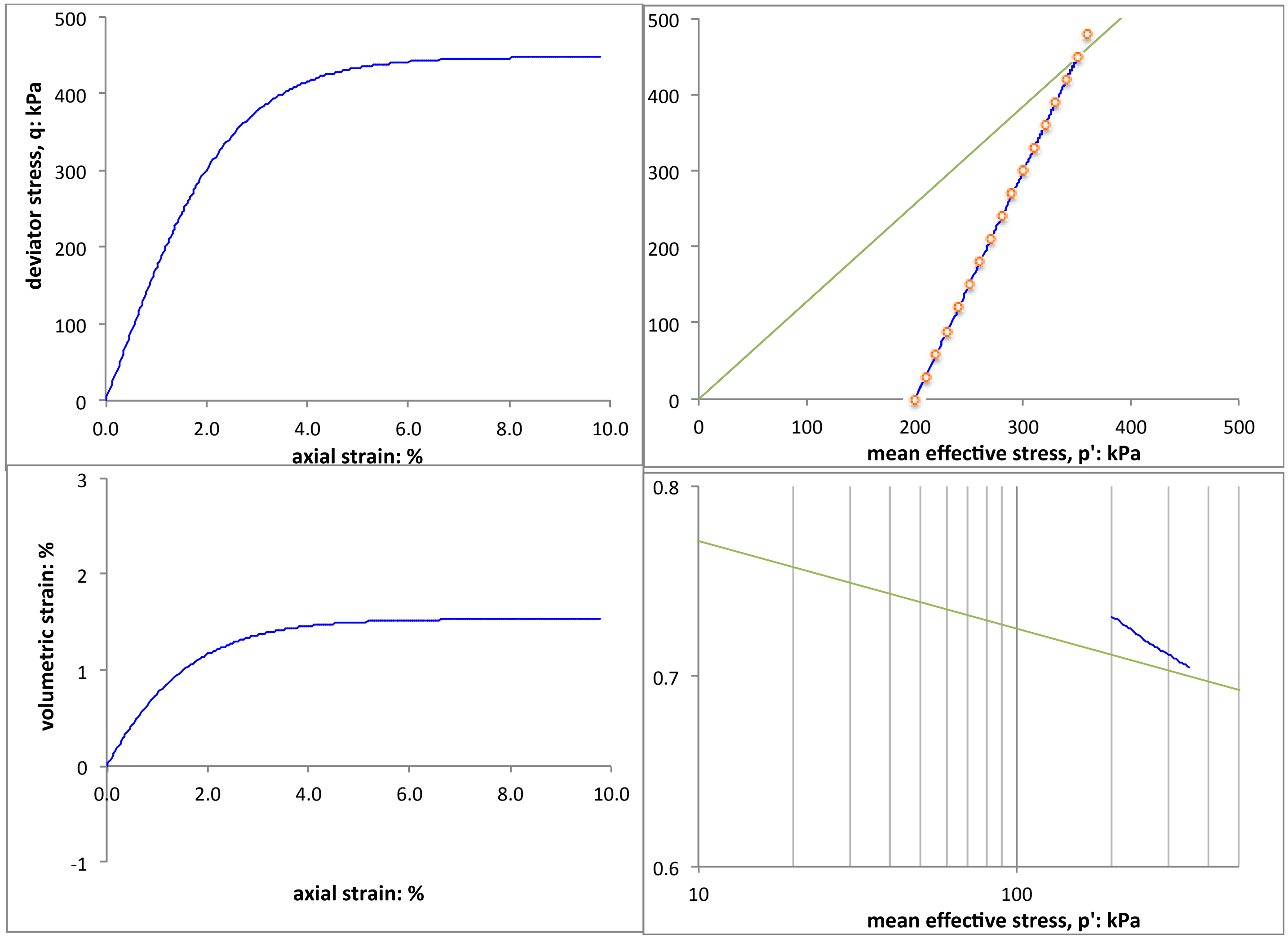
- Either make copy of undrained worksheet or duplicate on same sheet
  - Load path already defined on sheet (check dq/dp = 3)

- Add in column for…
  - Eta_dot (drained consistency condition)
- Modify
  - Step size
  - Calc of p_dot
  - Calc of eta…. $\eta_{j+1} = \eta_j + \dot{\eta}_j$
  - Increase number of steps to get 10% strain
- Graphs
  - Add volumetric strain plot & state plot
  - Add theoretical stress path to verify integration

# Drained OCC… xls modifications



# Computed soil response



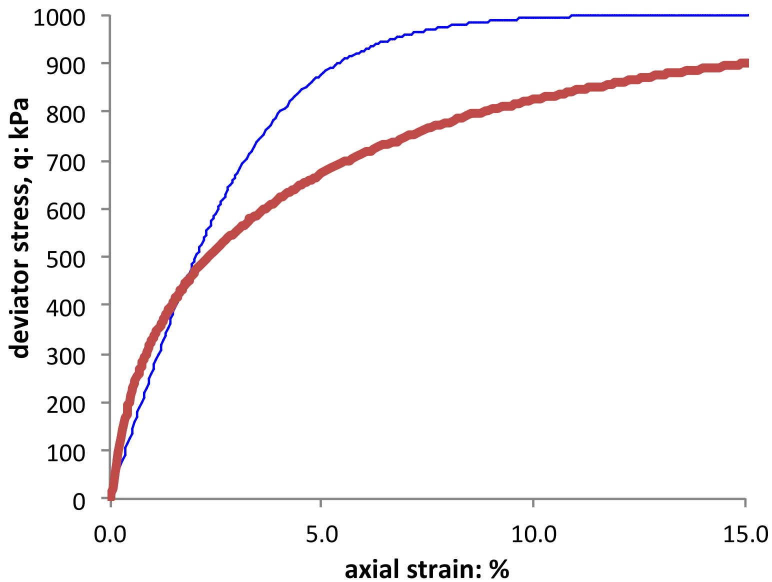
# Let us program….
# (validation then follows)

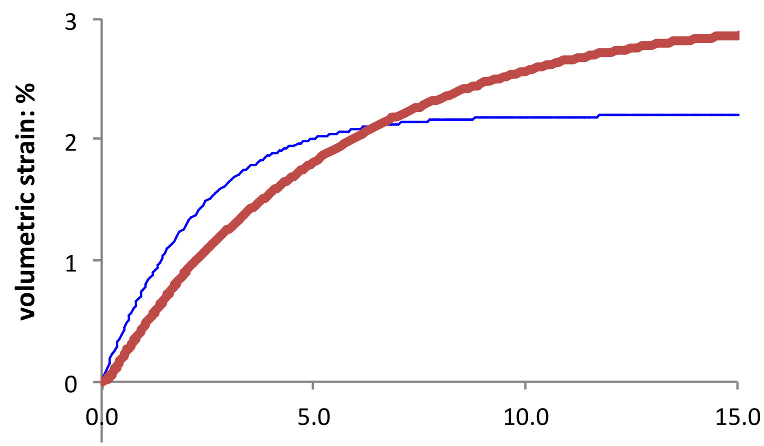
# Validation of OCC

- Find *TXL_Data.xls* in hand out
- Copy the sheet for test CID_682 to your OCC xls
- Plot ep1 vs q & epV vs ep1 on same plots as OCC
- $p_0$ = 500 kPa; fix plot scales
- Adjust OCC properties to fit data

# My fit to test

- $\Gamma = 0.93$
- $\lambda = 0.03$
- $\kappa = 0.007$
- $M = 1.2$



# Time to stop cheating…

- For isotropic test and Cam Clay…

  initial yield surface 'hardness' $p_c$ = $p_0$ / SpacingRatio

- Revert to void ratio form…

$$\ln(p_c) = \frac{\Gamma - e - \kappa \ln(p)}{\lambda - \kappa}$$

# BREAK for TEA

# What is wrong with Cam Clay…

- Back to *TXL_Data.xls* in hand out

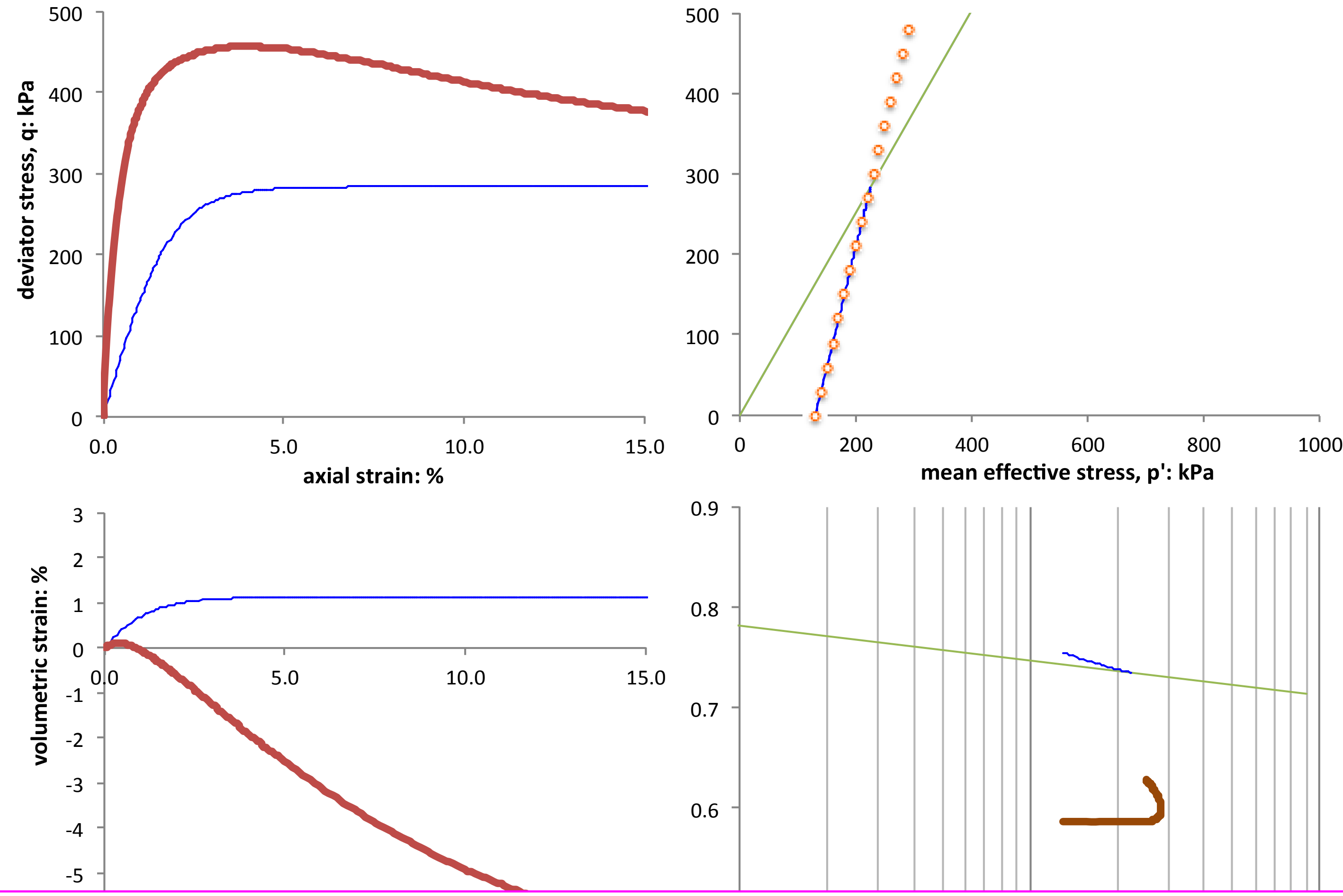
- Copy the sheet for test CID_667 to your OCC xls

- Switch data plot from test 682 to 667

- Set StepSize smaller…. 0.0001

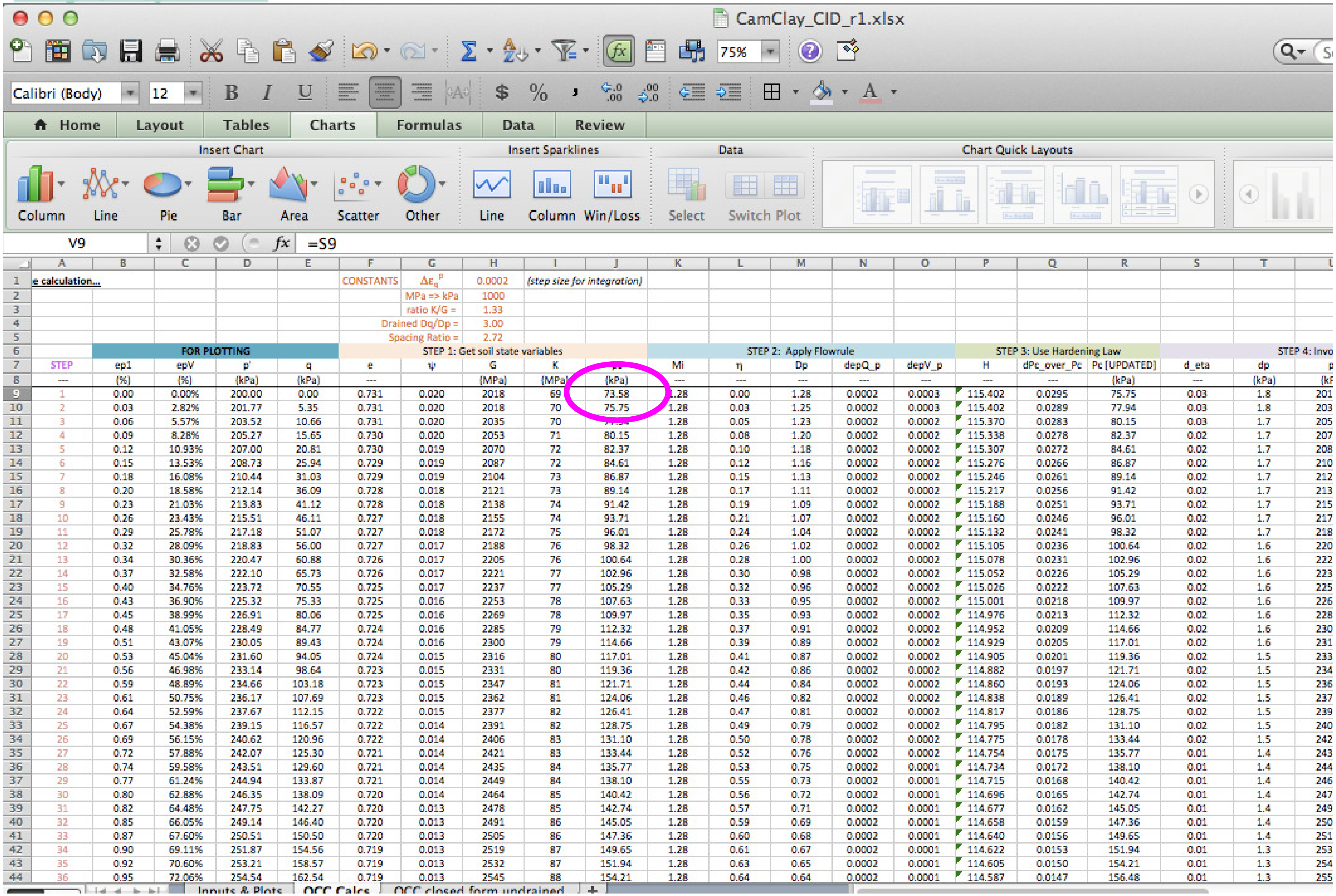
- Set OCC properties…
  - $P_0$ = 130 kPa
  - $\Gamma = 0.82$
  - $\lambda = 0.015$
  - $\kappa = 0.004$
  - $M = 1.26$

# Worksheet plots CID_667 imported



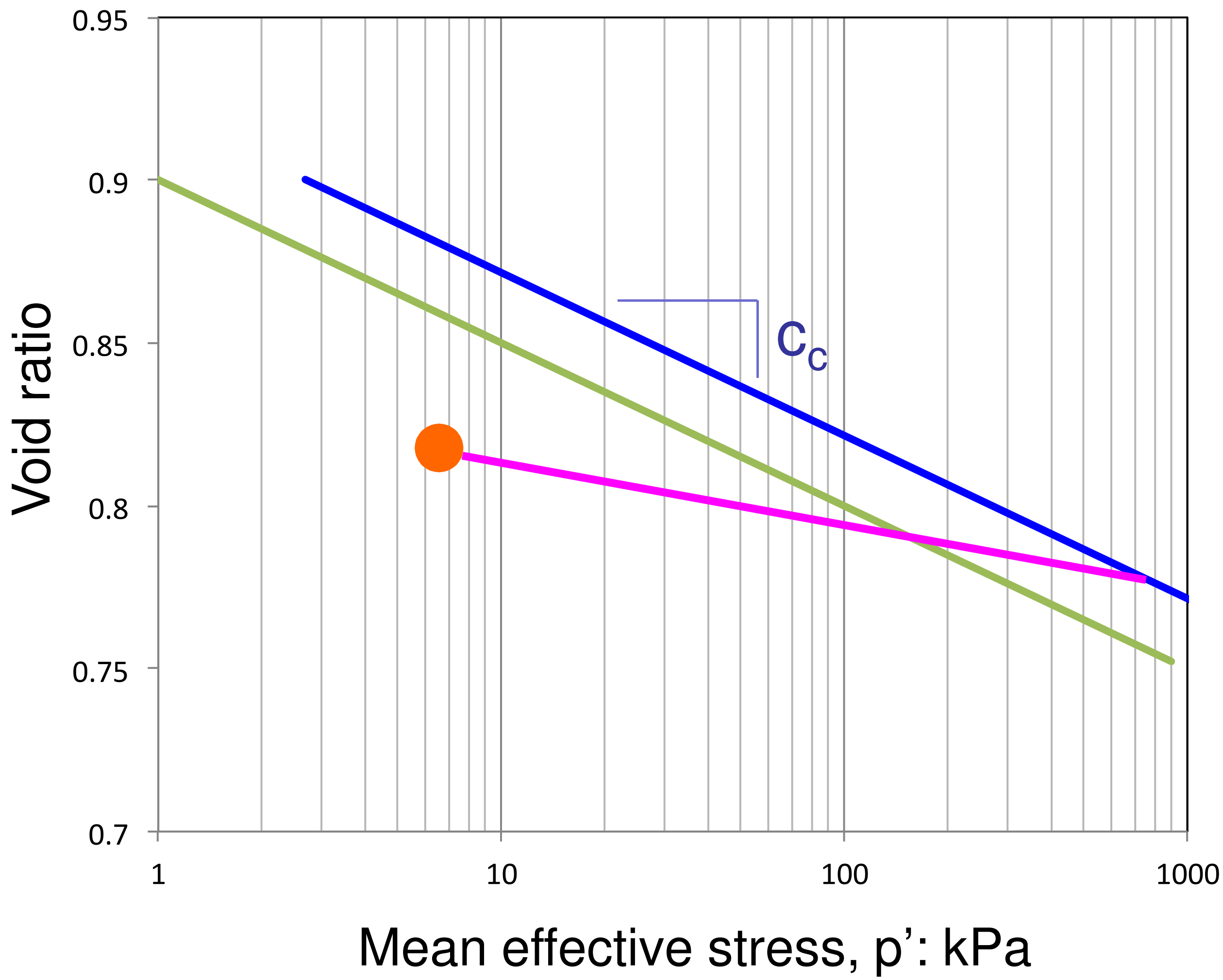

Now try and honour the measured initial void ratio in this test (by freezing method)... **SET: $e_0$ = 0.7**

# Drained OCC… xls modifications

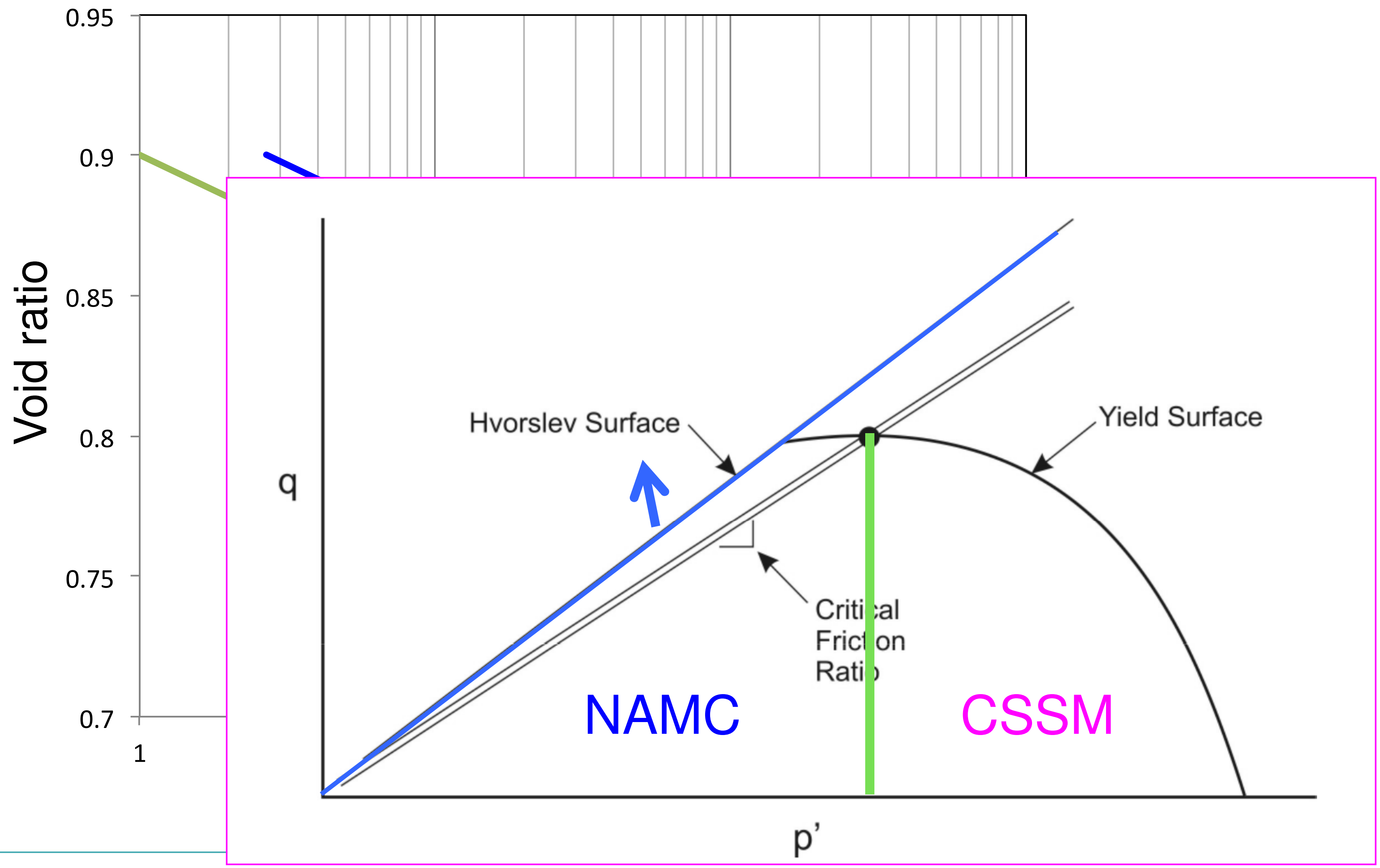



| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | e calculation... | | | | | CONSTANTS | $\Delta\varepsilon_q^p$ | 0.0002 | (step size for integration) | | | | | | | | | | | | |
| 2 | | | | | | | MPa => kPa | 1000 | | | | | | | | | | | | | |
| 3 | | | | | | | ratio K/G = | 1.33 | | | | | | | | | | | | | |
| 4 | | | | | | | Drained Dq/Dp = | 3.00 | | | | | | | | | | | | | |
| 5 | | | | | | | Spacing Ratio = | 2.72 | | | | | | | | | | | | | |
| 6 | | FOR PLOTTING | | | | STEP 1: Get soil state variables | | | | | STEP 2: Apply Flowrule | | | | | STEP 3: Use Hardening Law | | | | STEP 4: Invo | |
| 7 | STEP | ep1 | epV | p' | q | e | ψ | G | K | [illegible] | Mi | η | Dp | depQ_p | depV_p | H | dPc_over_Pc | Pc [UPDATED] | d_eta | dp | p |
| 8 | --- | (%) | (%) | (kPa) | (kPa) | --- | | (MPa) | (MPa) | (kPa) | --- | --- | --- | --- | --- | --- | --- | (kPa) | --- | (kPa) | (kP |
| 9 | 1 | 0.00 | 0.00% | 200.00 | 0.00 | 0.731 | 0.020 | 2018 | 69 | 73.58 | 1.28 | 0.00 | 1.28 | 0.0002 | 0.0003 | 115.402 | 0.0295 | 75.75 | 0.03 | 1.8 | 201 |
| 10 | 2 | 0.03 | 2.82% | 201.77 | 5.35 | 0.731 | 0.020 | 2018 | 70 | 75.75 | 1.28 | 0.03 | 1.25 | 0.0002 | 0.0003 | 115.402 | 0.0289 | 77.94 | 0.03 | 1.8 | 203 |
| 11 | 3 | 0.06 | 5.57% | 203.52 | 10.66 | 0.731 | 0.020 | 2035 | 70 | [illegible] | 1.28 | 0.05 | 1.23 | 0.0002 | 0.0002 | 115.370 | 0.0283 | 80.15 | 0.03 | 1.7 | 205 |
| 12 | 4 | 0.09 | 8.28% | 205.27 | 15.65 | 0.730 | 0.020 | 2053 | 71 | 80.15 | 1.28 | 0.08 | 1.20 | 0.0002 | 0.0002 | 115.338 | 0.0278 | 82.37 | 0.02 | 1.7 | 207 |
| 13 | 5 | 0.12 | 10.93% | 207.00 | 20.81 | 0.730 | 0.019 | 2070 | 72 | 82.37 | 1.28 | 0.10 | 1.18 | 0.0002 | 0.0002 | 115.307 | 0.0272 | 84.61 | 0.02 | 1.7 | 208 |
| 14 | 6 | 0.15 | 13.53% | 208.73 | 25.94 | 0.729 | 0.019 | 2087 | 72 | 84.61 | 1.28 | 0.12 | 1.16 | 0.0002 | 0.0002 | 115.276 | 0.0266 | 86.87 | 0.02 | 1.7 | 210 |
| 15 | 7 | 0.18 | 16.08% | 210.44 | 31.03 | 0.729 | 0.019 | 2104 | 73 | 86.87 | 1.28 | 0.15 | 1.13 | 0.0002 | 0.0002 | 115.246 | 0.0261 | 89.14 | 0.02 | 1.7 | 212 |
| 16 | 8 | 0.20 | 18.58% | 212.14 | 36.09 | 0.728 | 0.018 | 2121 | 73 | 89.14 | 1.28 | 0.17 | 1.11 | 0.0002 | 0.0002 | 115.217 | 0.0256 | 91.42 | 0.02 | 1.7 | 213 |
| 17 | 9 | 0.23 | 21.03% | 213.83 | 41.12 | 0.728 | 0.018 | 2138 | 74 | 91.42 | 1.28 | 0.19 | 1.09 | 0.0002 | 0.0002 | 115.188 | 0.0251 | 93.71 | 0.02 | 1.7 | 215 |
| 18 | 10 | 0.26 | 23.43% | 215.51 | 46.11 | 0.727 | 0.018 | 2155 | 74 | 93.71 | 1.28 | 0.21 | 1.07 | 0.0002 | 0.0002 | 115.160 | 0.0246 | 96.01 | 0.02 | 1.7 | 217 |
| 19 | 11 | 0.29 | 25.78% | 217.18 | 51.07 | 0.727 | 0.018 | 2172 | 75 | 96.01 | 1.28 | 0.24 | 1.04 | 0.0002 | 0.0002 | 115.132 | 0.0241 | 98.32 | 0.02 | 1.7 | 218 |
| 20 | 12 | 0.32 | 28.09% | 218.83 | 56.00 | 0.727 | 0.017 | 2188 | 76 | 98.32 | 1.28 | 0.26 | 1.02 | 0.0002 | 0.0002 | 115.105 | 0.0236 | 100.64 | 0.02 | 1.6 | 220 |
| 21 | 13 | 0.34 | 30.36% | 220.47 | 60.88 | 0.726 | 0.017 | 2205 | 76 | 100.64 | 1.28 | 0.28 | 1.00 | 0.0002 | 0.0002 | 115.078 | 0.0231 | 102.96 | 0.02 | 1.6 | 222 |
| 22 | 14 | 0.37 | 32.58% | 222.10 | 65.73 | 0.726 | 0.017 | 2221 | 77 | 102.96 | 1.28 | 0.30 | 0.98 | 0.0002 | 0.0002 | 115.052 | 0.0226 | 105.29 | 0.02 | 1.6 | 223 |
| 23 | 15 | 0.40 | 34.76% | 223.72 | 70.55 | 0.725 | 0.017 | 2237 | 77 | 105.29 | 1.28 | 0.32 | 0.96 | 0.0002 | 0.0002 | 115.026 | 0.0222 | 107.63 | 0.02 | 1.6 | 225 |
| 24 | 16 | 0.43 | 36.90% | 225.32 | 75.33 | 0.725 | 0.016 | 2253 | 78 | 107.63 | 1.28 | 0.33 | 0.95 | 0.0002 | 0.0002 | 115.001 | 0.0218 | 109.97 | 0.02 | 1.6 | 226 |
| 25 | 17 | 0.45 | 38.99% | 226.91 | 80.06 | 0.725 | 0.016 | 2269 | 78 | 109.97 | 1.28 | 0.35 | 0.93 | 0.0002 | 0.0002 | 114.976 | 0.0213 | 112.32 | 0.02 | 1.6 | 228 |
| 26 | 18 | 0.48 | 41.05% | 228.49 | 84.77 | 0.724 | 0.016 | 2285 | 79 | 112.32 | 1.28 | 0.37 | 0.91 | 0.0002 | 0.0002 | 114.952 | 0.0209 | 114.66 | 0.02 | 1.6 | 230 |
| 27 | 19 | 0.51 | 43.07% | 230.05 | 89.43 | 0.724 | 0.016 | 2300 | 79 | 114.66 | 1.28 | 0.39 | 0.89 | 0.0002 | 0.0002 | 114.929 | 0.0205 | 117.01 | 0.02 | 1.6 | 231 |
| 28 | 20 | 0.53 | 45.04% | 231.60 | 94.05 | 0.724 | 0.015 | 2316 | 80 | 117.01 | 1.28 | 0.41 | 0.87 | 0.0002 | 0.0002 | 114.905 | 0.0201 | 119.36 | 0.02 | 1.5 | 233 |
| 29 | 21 | 0.56 | 46.98% | 233.14 | 98.64 | 0.723 | 0.015 | 2331 | 80 | 119.36 | 1.28 | 0.42 | 0.86 | 0.0002 | 0.0002 | 114.882 | 0.0197 | 121.71 | 0.02 | 1.5 | 234 |
| 30 | 22 | 0.59 | 48.89% | 234.66 | 103.18 | 0.723 | 0.015 | 2347 | 81 | 121.71 | 1.28 | 0.44 | 0.84 | 0.0002 | 0.0002 | 114.860 | 0.0193 | 124.06 | 0.02 | 1.5 | 236 |
| 31 | 23 | 0.61 | 50.75% | 236.17 | 107.69 | 0.723 | 0.015 | 2362 | 81 | 124.06 | 1.28 | 0.46 | 0.82 | 0.0002 | 0.0002 | 114.838 | 0.0189 | 126.41 | 0.02 | 1.5 | 237 |
| 32 | 24 | 0.64 | 52.59% | 237.67 | 112.15 | 0.722 | 0.015 | 2377 | 82 | 126.41 | 1.28 | 0.47 | 0.81 | 0.0002 | 0.0002 | 114.817 | 0.0186 | 128.75 | 0.02 | 1.5 | 239 |
| 33 | 25 | 0.67 | 54.38% | 239.15 | 116.57 | 0.722 | 0.014 | 2391 | 82 | 128.75 | 1.28 | 0.49 | 0.79 | 0.0002 | 0.0002 | 114.795 | 0.0182 | 131.10 | 0.02 | 1.5 | 240 |
| 34 | 26 | 0.69 | 56.15% | 240.62 | 120.96 | 0.722 | 0.014 | 2406 | 83 | 131.10 | 1.28 | 0.50 | 0.78 | 0.0002 | 0.0002 | 114.775 | 0.0178 | 133.44 | 0.02 | 1.5 | 242 |
| 35 | 27 | 0.72 | 57.88% | 242.07 | 125.30 | 0.721 | 0.014 | 2421 | 83 | 133.44 | 1.28 | 0.52 | 0.76 | 0.0002 | 0.0002 | 114.754 | 0.0175 | 135.77 | 0.01 | 1.4 | 243 |
| 36 | 28 | 0.74 | 59.58% | 243.51 | 129.60 | 0.721 | 0.014 | 2435 | 84 | 135.77 | 1.28 | 0.53 | 0.75 | 0.0002 | 0.0001 | 114.734 | 0.0172 | 138.10 | 0.01 | 1.4 | 244 |
| 37 | 29 | 0.77 | 61.24% | 244.94 | 133.87 | 0.721 | 0.014 | 2449 | 84 | 138.10 | 1.28 | 0.55 | 0.73 | 0.0002 | 0.0001 | 114.715 | 0.0168 | 140.42 | 0.01 | 1.4 | 246 |
| 38 | 30 | 0.80 | 62.88% | 246.35 | 138.09 | 0.720 | 0.014 | 2464 | 85 | 140.42 | 1.28 | 0.56 | 0.72 | 0.0002 | 0.0001 | 114.696 | 0.0165 | 142.74 | 0.01 | 1.4 | 247 |
| 39 | 31 | 0.82 | 64.48% | 247.75 | 142.27 | 0.720 | 0.013 | 2478 | 85 | 142.74 | 1.28 | 0.57 | 0.71 | 0.0002 | 0.0001 | 114.677 | 0.0162 | 145.05 | 0.01 | 1.4 | 249 |
| 40 | 32 | 0.85 | 66.05% | 249.14 | 146.40 | 0.720 | 0.013 | 2491 | 86 | 145.05 | 1.28 | 0.59 | 0.69 | 0.0002 | 0.0001 | 114.658 | 0.0159 | 147.36 | 0.01 | 1.4 | 250 |
| 41 | 33 | 0.87 | 67.60% | 250.51 | 150.50 | 0.720 | 0.013 | 2505 | 86 | 147.36 | 1.28 | 0.60 | 0.68 | 0.0002 | 0.0001 | 114.640 | 0.0156 | 149.65 | 0.01 | 1.4 | 251 |
| 42 | 34 | 0.90 | 69.11% | 251.87 | 154.56 | 0.719 | 0.013 | 2519 | 87 | 149.65 | 1.28 | 0.61 | 0.67 | 0.0002 | 0.0001 | 114.622 | 0.0153 | 151.94 | 0.01 | 1.3 | 253 |
| 43 | 35 | 0.92 | 70.60% | 253.21 | 158.57 | 0.719 | 0.013 | 2532 | 87 | 151.94 | 1.28 | 0.63 | 0.65 | 0.0002 | 0.0001 | 114.605 | 0.0150 | 154.21 | 0.01 | 1.3 | 254 |
| 44 | 36 | 0.95 | 72.06% | 254.54 | 162.54 | 0.719 | 0.013 | 2545 | 88 | 154.21 | 1.28 | 0.64 | 0.64 | 0.0002 | 0.0001 | 114.587 | 0.0147 | 156.48 | 0.01 | 1.3 | 255 |

# What is going on ?



# Comments on Cam Clay



## Summary

- OCC is beautiful mathematical construct
- Validates to real soil under very limited range of conditions
- OCC cannot deal with nearly everything encountered in practice
- Situation ~ 1975: " *Interesting, but CSSM a dead end* "

**GAME CHANGER….**

➡ **1985: State Parameter** (Parry, 1958)

➡ **1993: NorSand**