Developing confidence in critical state soil mechanics

3. Theory of Original Cam Clay (OCC)

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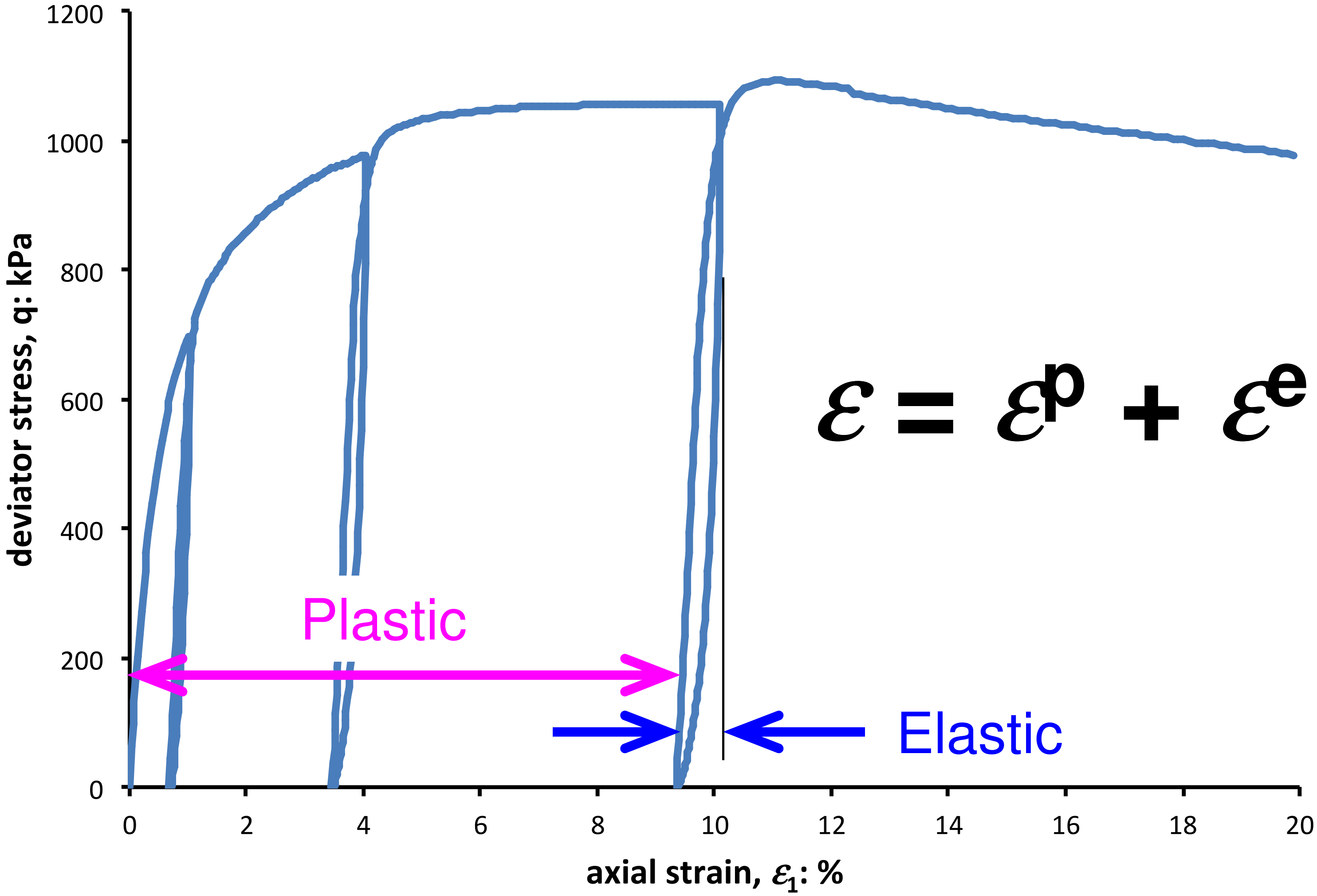
Philosophy

- Models
  - Descriptive (“curve fitting”)
    - *Can be accurate for a stress-path but no insight*
  - Idealized
    - *Known and consistent physics*
    - *General*
    - *Predictive power*

- Adequacy
  - useful models do not have to be perfect
  - useful models <u>must</u> capture salient aspects
  - want more than “strength”…

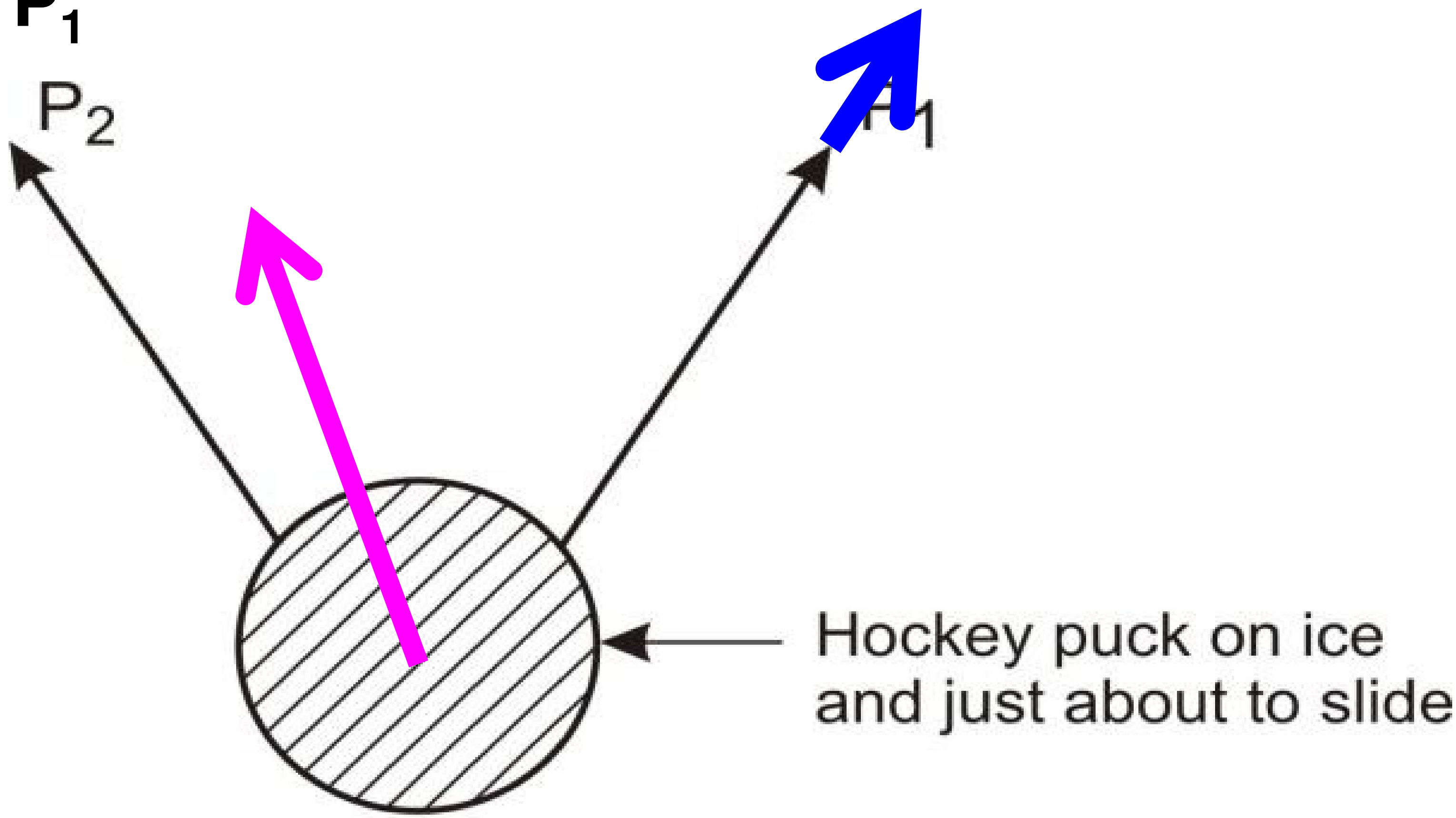
…all critical state models are idealized

Soil behaviour in triaxial compression

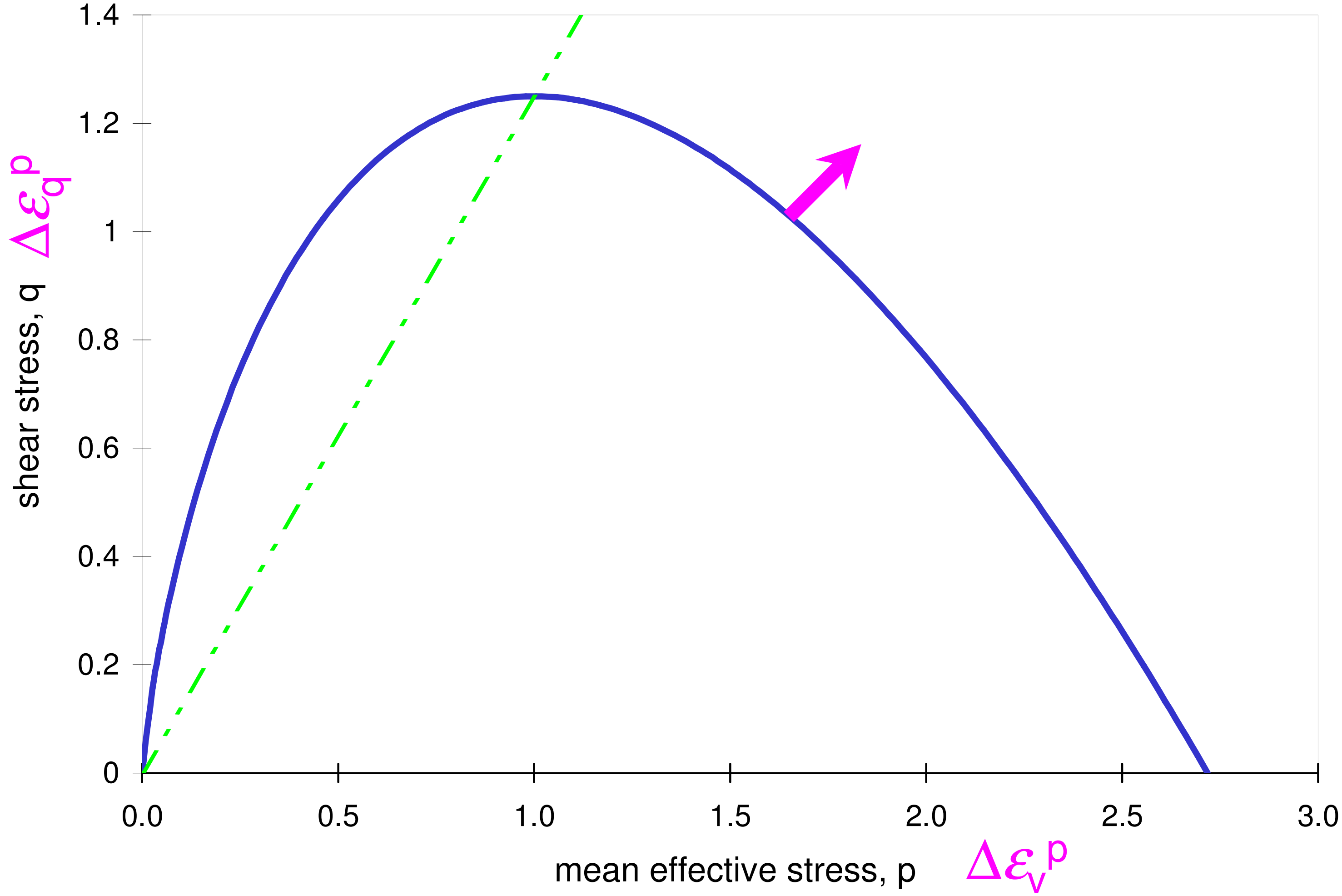


Plasticity affects which way soil moves…

$P_2 > P_1$



“Normality” = *Associated Flow Rule*



Work Hardening (Softening) Plasticity

- Yield surface in stress space
  - defines the elastic domain
  - cannot go outside it

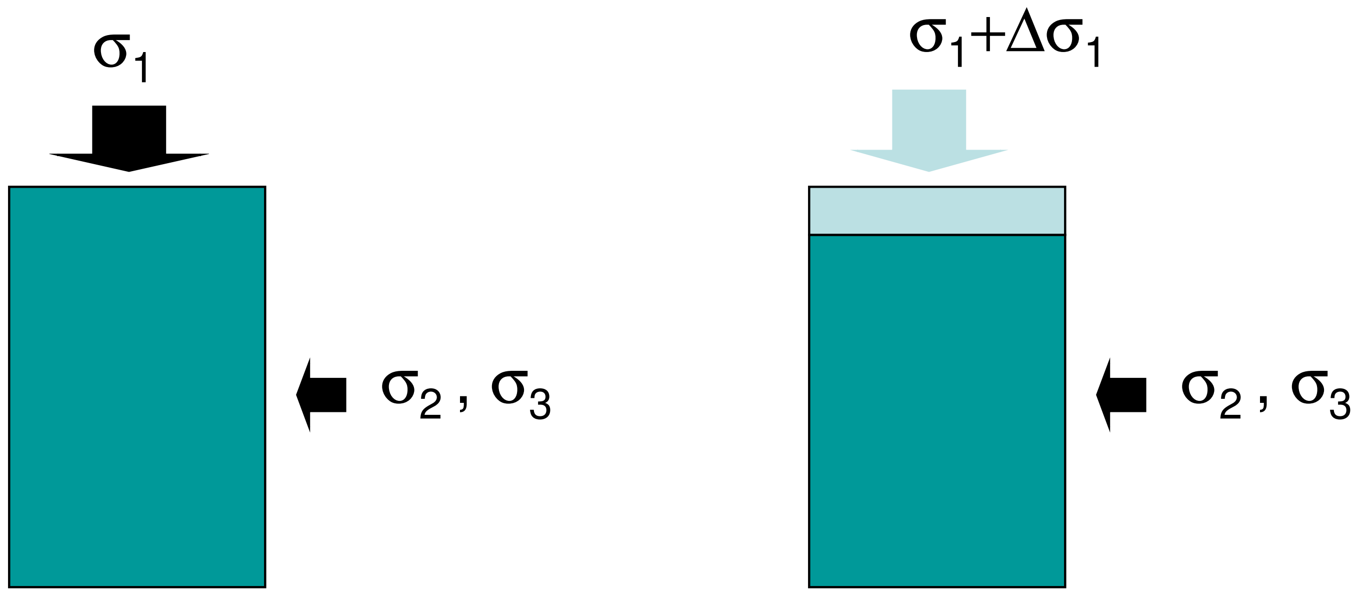
- Flowrule (plastic potential)
  - direction (ratio) of the plastic strain increments

- Hardening rule
  - how the yield surface evolves with *plastic strain*

- Elasticity
  - Both within and on yield surface
  - Constant ν seems appropriate, G & K = f(e,σ)

Derive the OCC flowrule

Consider a unit volume element of soil loaded by a strain increment...



Work: $\bar{\sigma}_q \dot{\varepsilon}_q + \bar{\sigma}_m \dot{\varepsilon}_v$

Plastic work dissipated by soil skeleton

Use elastic-plastic strain decomposition: $$\dot{W}^p = \dot{W} - \dot{W}^e = q\dot{\varepsilon}_q^p + p\dot{\varepsilon}_v^p$$

(1) Divide this plastic work rate first by the mean effective stress (to make it dimensionless);

(2) Divide by the plastic shear strain increment (normalized rate of working per unit plastic distortion of the soil)...

$$\Rightarrow \quad \frac{\dot{W}^p}{p\dot{\varepsilon}_q^p} = \frac{q}{p} + \frac{\dot{\varepsilon}_v^p}{\dot{\varepsilon}_q^p} = \boxed{\eta + D^p}$$

"universally acknowledged truth" (no constitutive model)

The OCC idealization

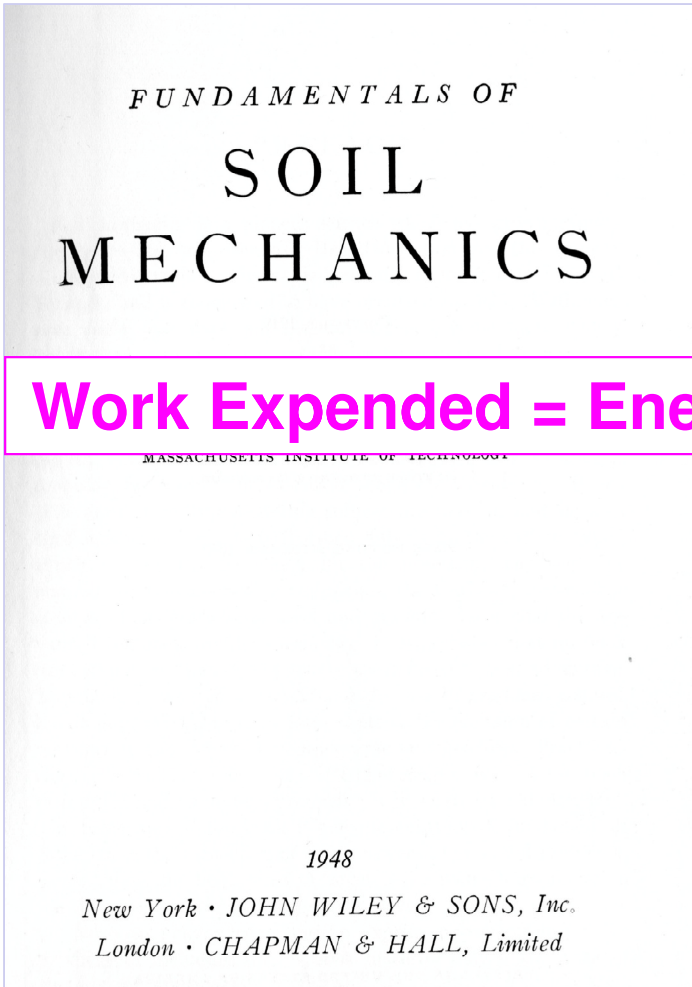
Postulate:(model) $\dot{W}^p = M\ p\left|\dot{\varepsilon}_q^p\right| \quad \Rightarrow \quad \dfrac{\dot{W}^p}{\overline{p}\,\dot{\varepsilon}_q^p} = M$

Universally known truth... $\dfrac{\dot{W}^p}{p\,\dot{\varepsilon}_q^p} = D^p + \eta$

$$D^p = M - \eta$$

Flowrule (Stress-Dilatancy)

Two components to soil strength (1948 - 50)



FUNDAMENTALS OF

SOIL
MECHANICS

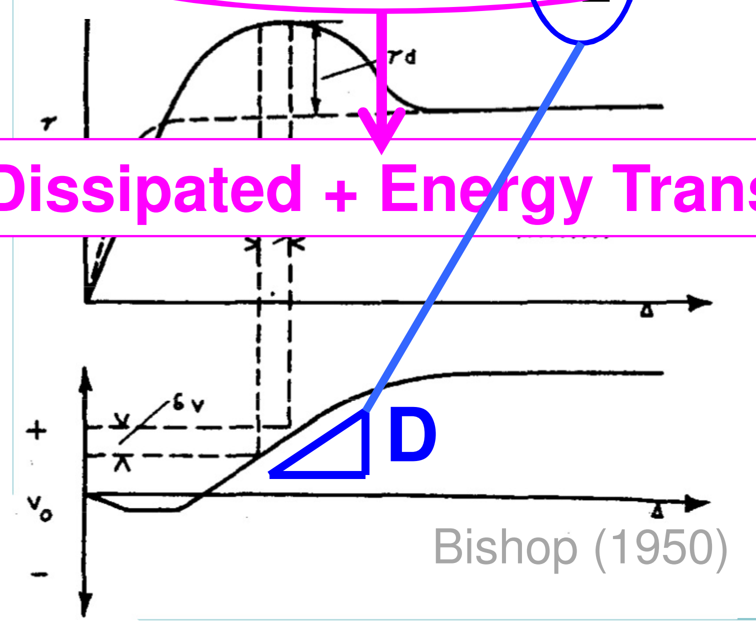
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

1948

New York · JOHN WILEY & SONS, Inc.
London · CHAPMAN & HALL, Limited

$$\tan(\phi) = \tan(\phi_f) + \frac{\delta V}{\delta \Delta}$$

Work Expended = Energy Dissipated + Energy Transferred



Derivation of OCC yield surface

From normality…

$$\frac{\dot{p}}{p} + \frac{\dot{\eta}}{D^p + \eta} = 0$$

From plastic working...

$$M = D^p + \eta$$

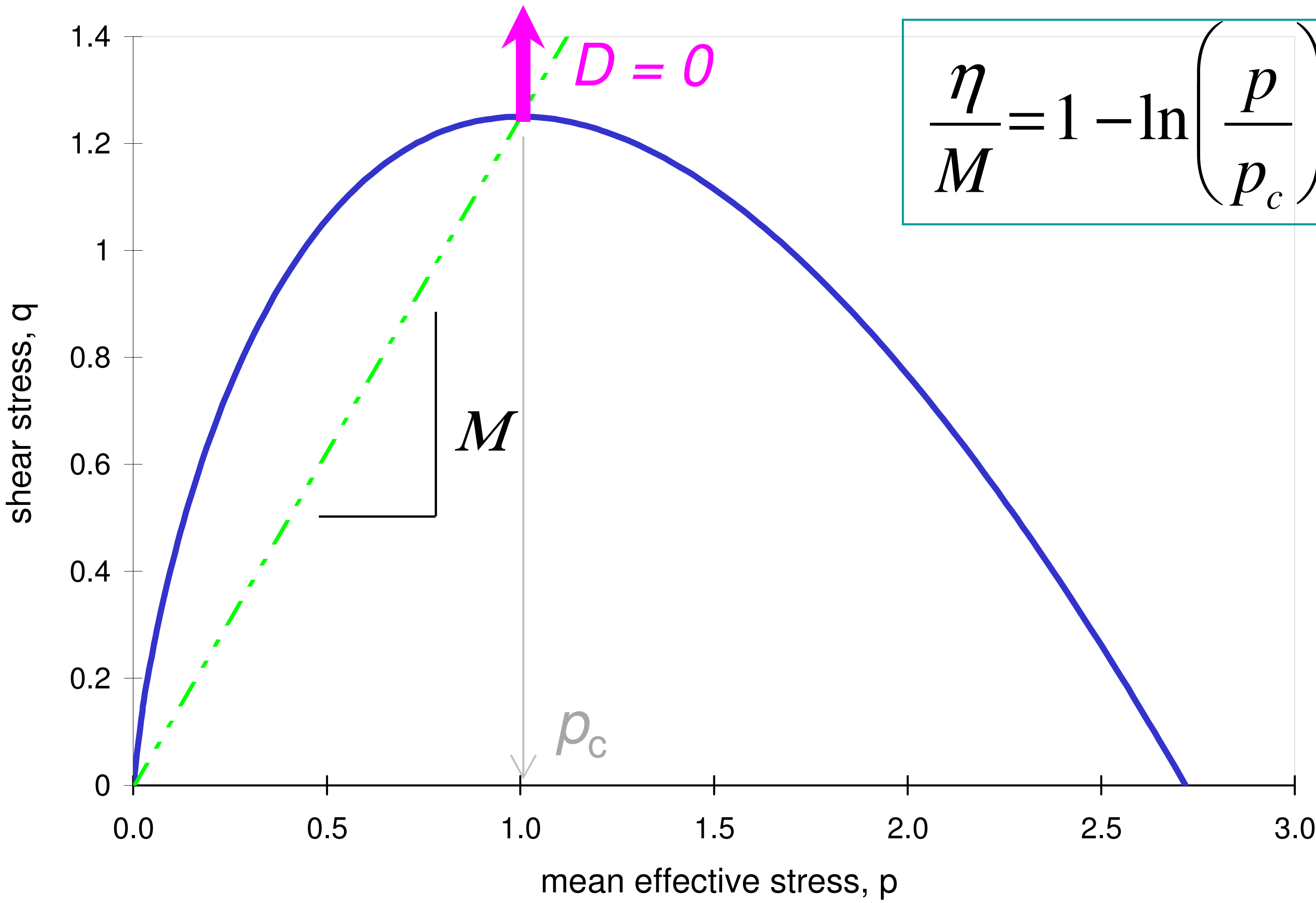
Combine…

$$\frac{\dot{p}}{p} + \frac{\dot{\eta}}{M} = 0$$

And integrate...

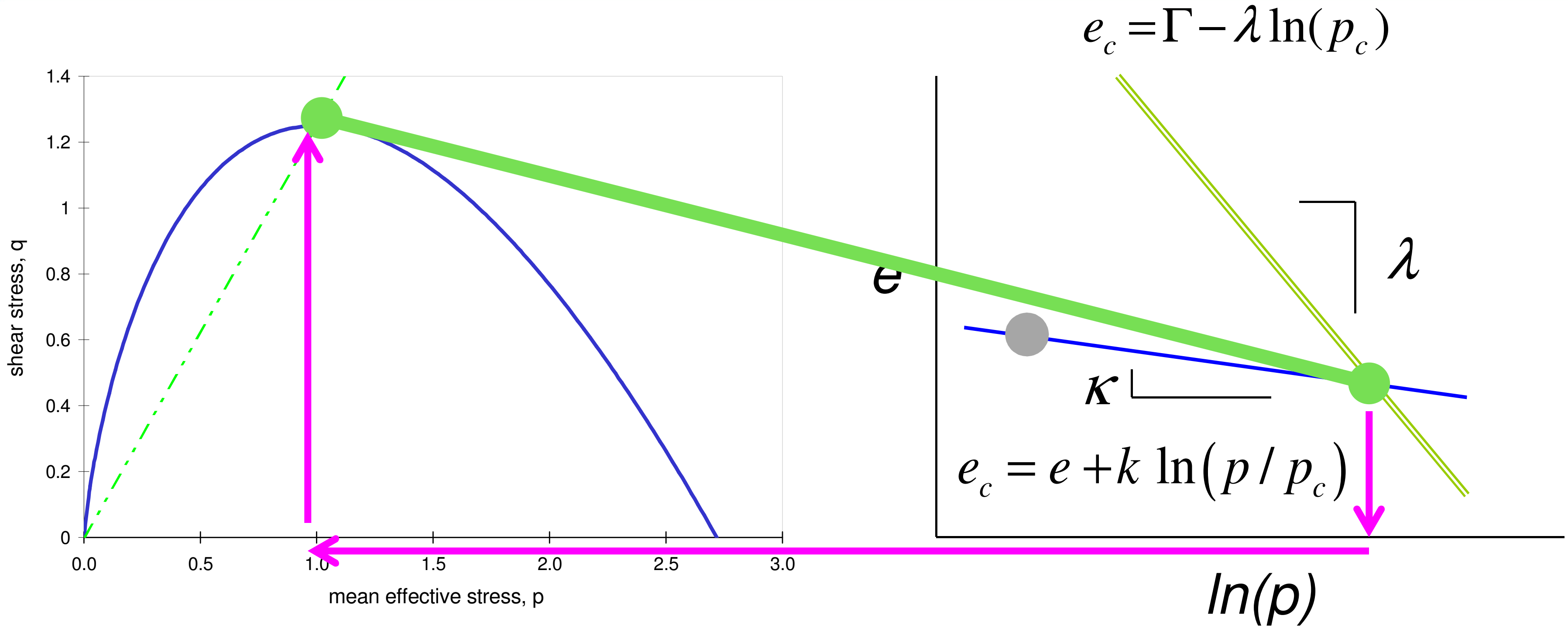
$$\ln(p) + \frac{\eta}{M} = C$$

Choice of integration term "C"



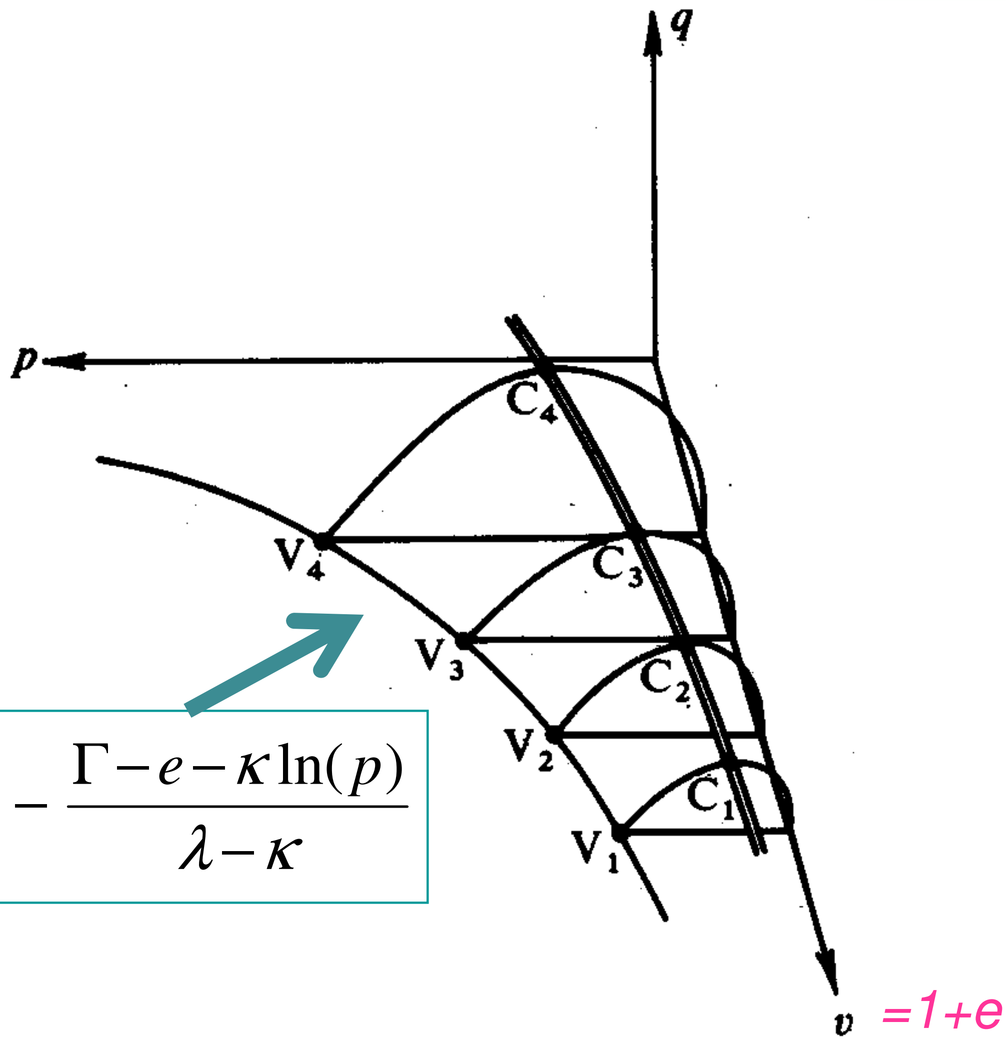
$$\frac{\eta}{M} = 1 - \ln\left(\frac{p}{p_c}\right)$$

How does void ratio control strength ?



$$1 - \frac{\eta}{M} = \ln(p) - \frac{\Gamma - e - \kappa \ln(p)}{\lambda - \kappa}$$

“state boundary surface”



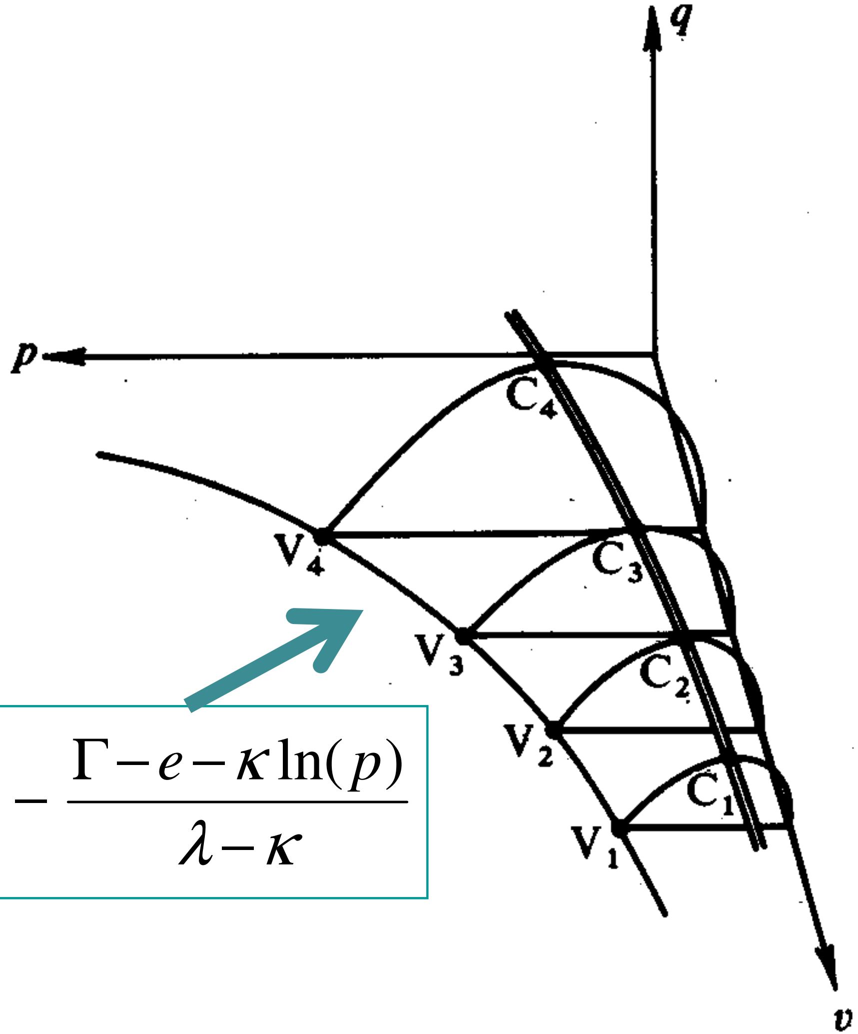
$$1-\frac{\eta}{M}=\ln(p)-\frac{\Gamma-e-\kappa\ln(p)}{\lambda-\kappa}$$

OCC – the equations

- Yield surface: $$\frac{\eta}{M} = 1 - \ln\left(\frac{p}{p_c}\right)$$ Normality & Work
- Flowrule: $$D^p = M - \eta$$ Work
- Hardening: $$\ln(p_c) = \frac{\Gamma - e - \kappa \ln(p)}{\lambda - \kappa}$$ CSL
- Elasticity: $$K = p(1+e)/\kappa \quad G = \infty$$

Only 4 soil properties… Γ, λ, κ, M

Barking mad view !



$$1-\frac{\eta}{M}=\ln(p)-\frac{\Gamma-e-\kappa\ln(p)}{\lambda-\kappa}$$

$=1+e$

Let us do hardening properly…

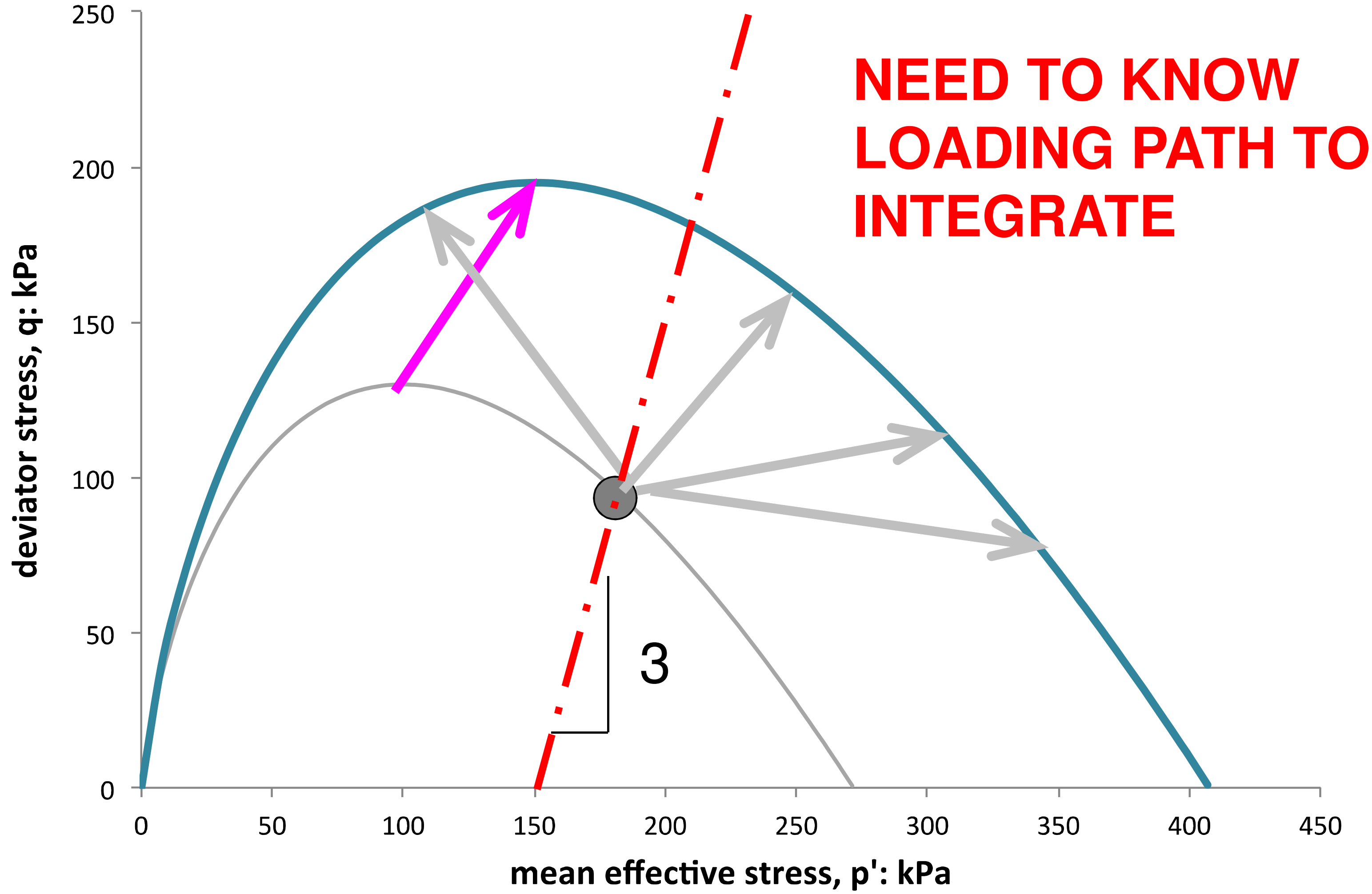
- Take the hardening law: $(\lambda - \kappa)\ln(p_c) = \Gamma - e - \kappa\ln(p)$
- Differentiate…………… $(\lambda - \kappa)\left(\frac{\dot{p}_c}{p_c}\right) = -\dot{e} - \kappa\frac{\dot{p}}{p}$
- Strain to void ratio…… $(1+e)\dot{\varepsilon}_v^p = -\dot{e}^p$
- **PLASTIC HARDENING:**

$$\frac{\dot{p}_c}{p_c} = \frac{1+e}{\lambda - \kappa}\dot{\varepsilon}_v^p$$

H

See J&B Appendix H

Consistency condition



Undrained behaviour

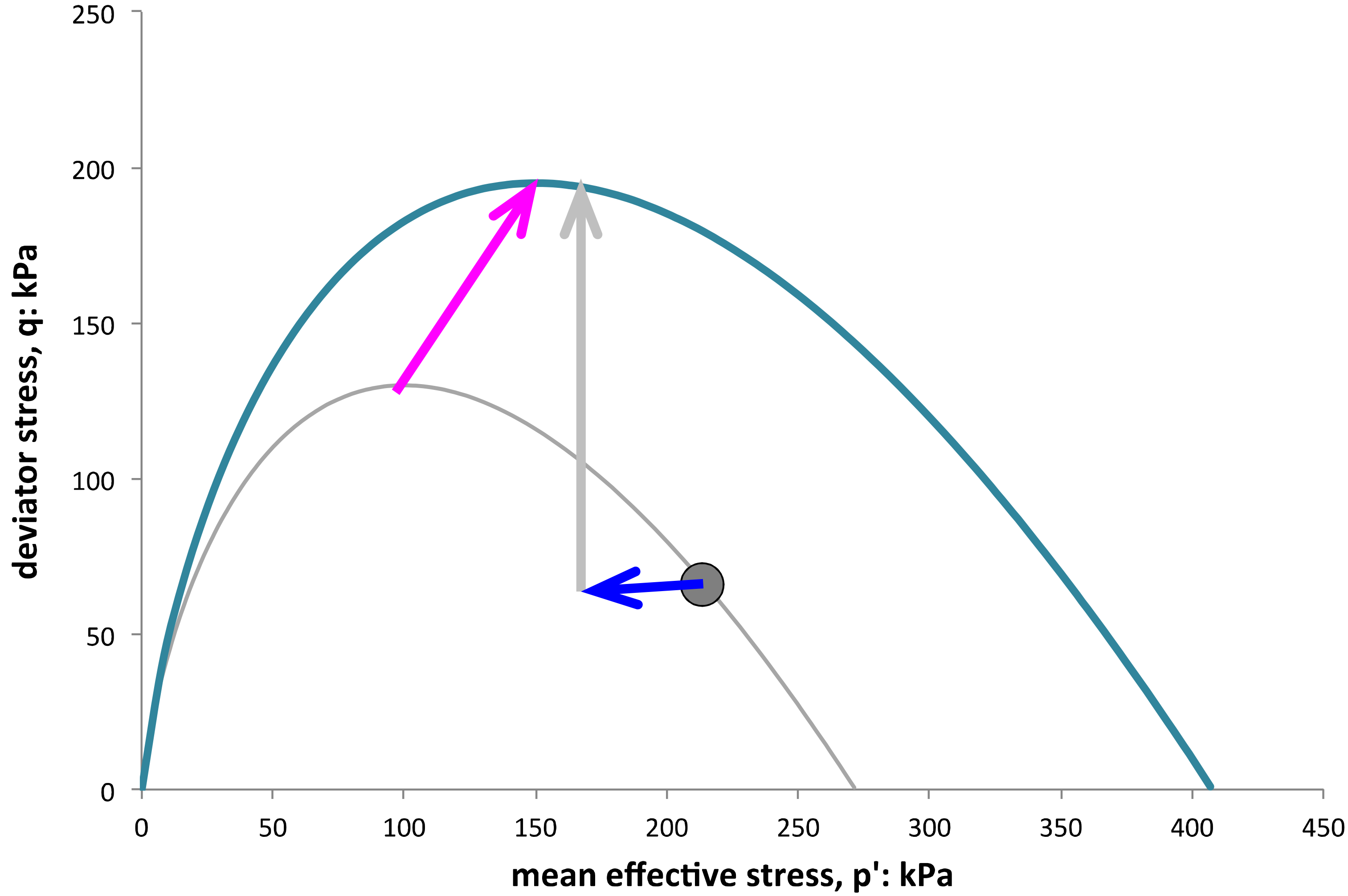
Undrained: impose e = constant (= no volumetric strain)

$$\dot{\varepsilon}_v = \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^p = 0 \quad \Rightarrow \dot{\varepsilon}_v^e = -\dot{\varepsilon}_v^p$$

$$\dot{u} = -\dot{p} = K\,\dot{\varepsilon}_v^p$$

- Undrained is a boundary condition, not a special soil behaviour
- Soil properties are unchanged by changes in boundary conditions
- Poisson's ratio ≠ 0.5 ! *(test data for soils ν ~ 0.2)*

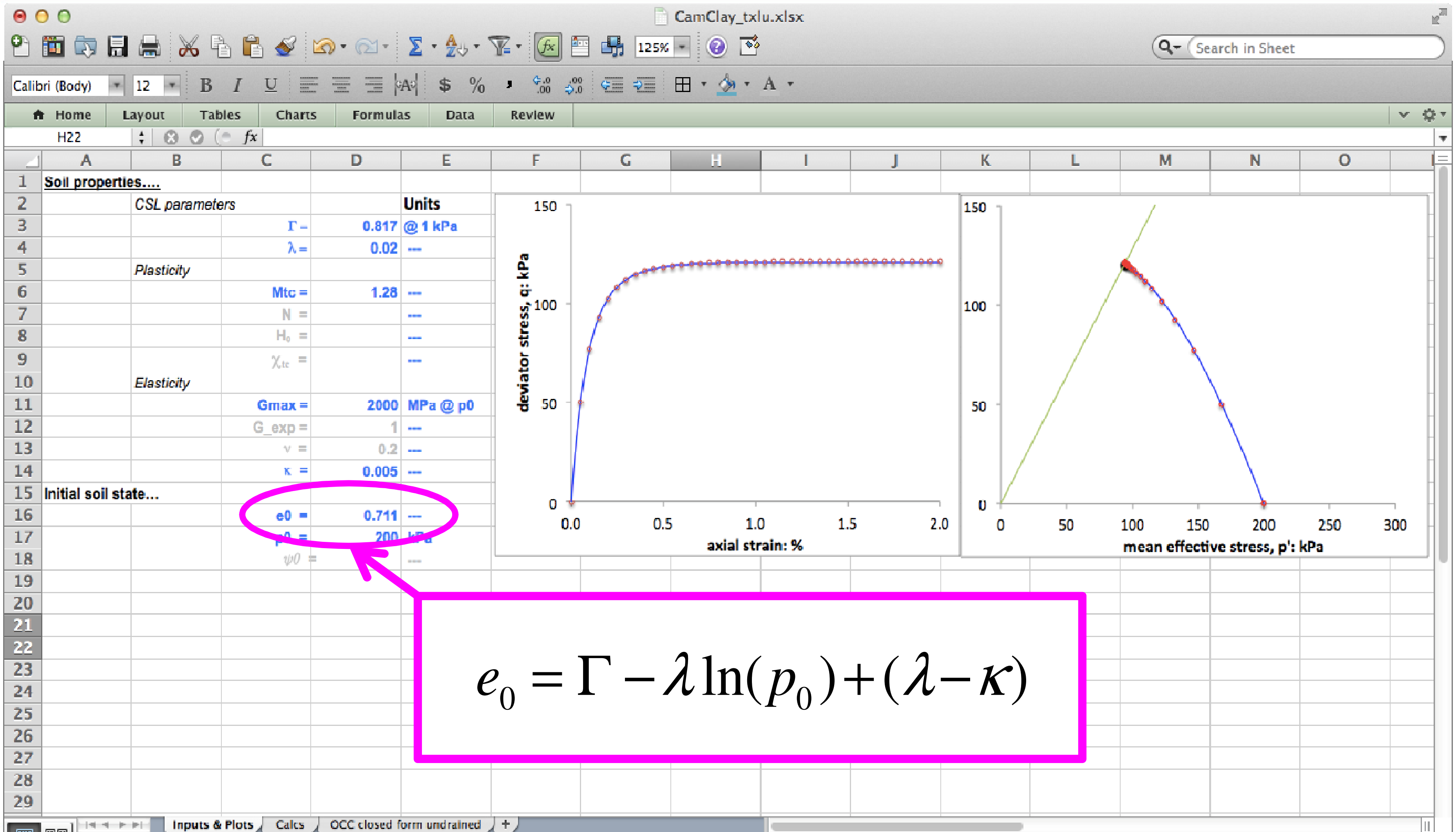
Undrained yielding



Comments on OCC

- Looking at triaxial today but OCC ideas generalize to 3D
- Can include more sophisticated CSL idealizations
- Trivial to add elastic shear modulus
- Start of modern era of soil mechanics
  - Links void ratio and stress level into soil behaviour
  - Works OK for soils in limited range of situations
  - Taught worldwide
  - Closed form solutions for two specific paths
- For history buffs see pdf in the handout on plasticity development

Initial void ratio for isotropic test…



$$e_0 = \Gamma - \lambda \ln(p_0) + (\lambda - \kappa)$$

Now its your turn… OCC in CIU txl test