VGS Workshop
critical state soil mechanics
16/17 January, 2015

Copy these files from memstick to your computer…

/VGS_CSSM_Jan2015

- **/data_and_progs** stuff you will use today
- **/notes_and_refs** pdf's to mostly save you writing

Developing confidence in critical state soil mechanics

1. Stress measures & Euler Integration

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Learning goals

- CSSM is more than Cam Clay
  - Complete, quantitative framework of soil behaviour
  - Soil properties are independent of e, σ
  - Not locked into semi-log CSL
  - *It is simple*

- Use in practice
  - Adjust laboratory data for disturbance
  - Site-specific calibration of CPT
  - FE modelling can be done, but not part of this workshop

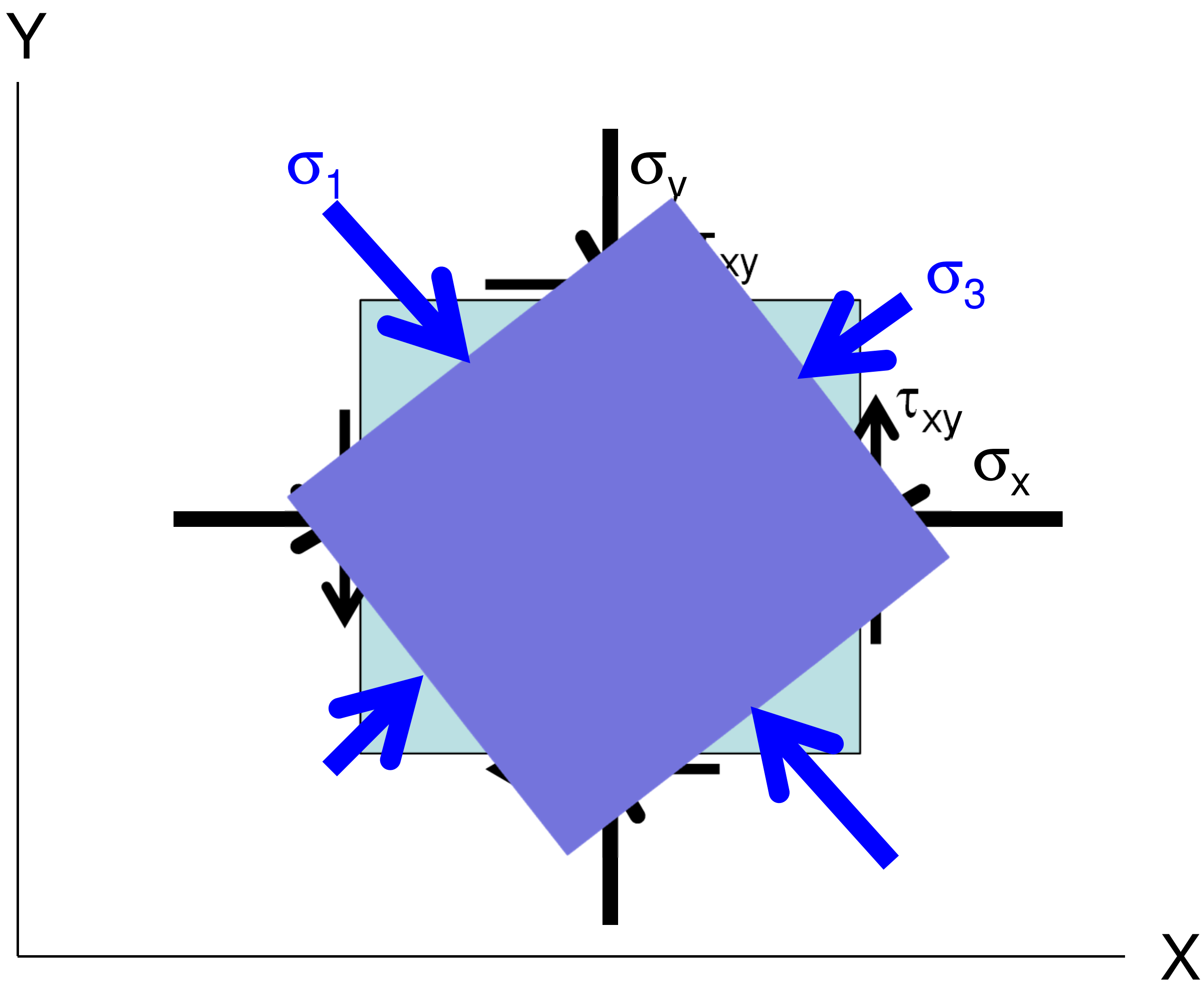
- More general
  - Affect the way you look at soil as an engineer
  - State parameter rules geotechnics…

Before we talk about soil models…

- 1: You do not have a free choice on stress and strain measures…
  - Why ?
  - What measures to use

- 2: Numerical integration
  - All proper plasticity models are written in terms of strain increment – basic requirement of mechanics
  - Need numerical integration for engineering
  - Integration principles

Principal stresses



All stresses 'effective' unless otherwise stated

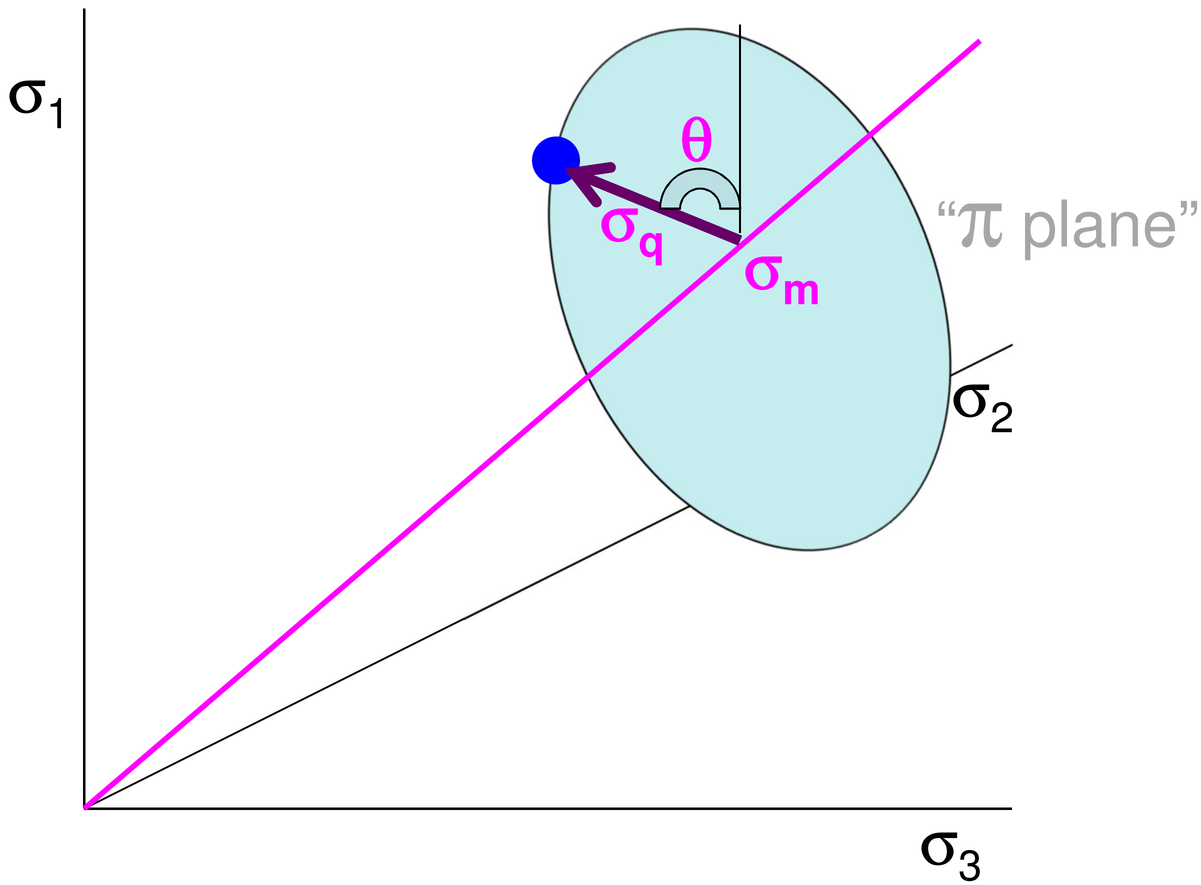
Stress and strain invariants (triaxial)

- Desire to make equations independent of measurement "frame"
  - Soils change both volume and shape
  - Soils "frictional" with strength depending on confining stress
  - Want to distinguish each aspect to allow clarity in understanding

- Stress invariants
  - Mean stress (change in volume): p or $\sigma_m = (\sigma_1 + 2\sigma_3) / 3$
  - Deviator stress (change in shape): q or $\sigma_q = (\sigma_1 - \sigma_3)$
  - Proportion of σ_2 : Bishop's "*b*" or Lode angle (θ)

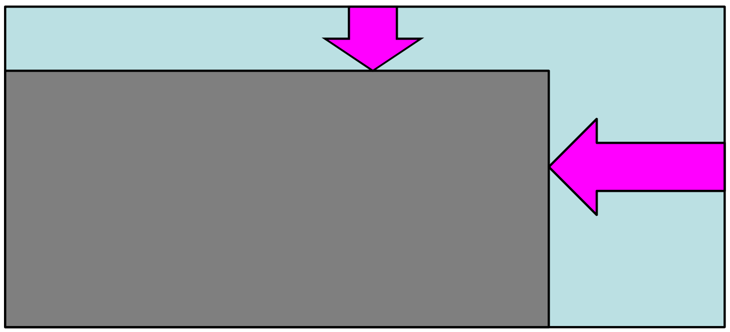
- Strain invariants
  - Volumetric strain: ε_v or $\varepsilon_m = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)$
  - Deviatoric strain: ε_q or $\varepsilon_\gamma = {}^2/_3\,(\varepsilon_1 - \varepsilon_3)$

$\sigma_1, \sigma_2, \sigma_3 = \sigma_m, \sigma_q, \theta$



Stress & strain measures

WORK DONE = Force x Distance Moved



INCREMENTAL WORK PER UNIT VOLUME:

$$\Delta W = \sigma_1 \Delta\varepsilon_1 + \sigma_2 \Delta\varepsilon_2 + \sigma_3 \Delta\varepsilon_3$$

Why restriction on choice of invariants

Plasticity: Materials dissipate *work* during *irrecoverable* straining.

Model validity requires getting work correct…

$$\sigma_1 \Delta\varepsilon_1 + \sigma_2 \Delta\varepsilon_2 + \sigma_3 \Delta\varepsilon_3 = \sigma_m \Delta\varepsilon_v + \sigma_q \Delta\varepsilon_q$$

…for TXL $= p\, \Delta\varepsilon_v + q\, \Delta\varepsilon_q$

Must use "work conjugate" stress & strain invariants

Work conjugate for triaxial compression

- Isotropic component
  - $\sigma_m = p = (\sigma_1 + 2\,\sigma_3) / 3$
  - $\varepsilon_v = (\varepsilon_1 + 2\,\varepsilon_3)$
- Deviatoric component
  - $\sigma_q = q = (\sigma_1 - \sigma_3)$
  - $\varepsilon_q = {}^2/_3(\varepsilon_1 - \varepsilon_3)$
- Lode angle
  - $\theta = 30$ deg

Strain decomposition (linear)

- $\varepsilon_v = \varepsilon_v^e + \varepsilon_v^p$

- $\varepsilon_q = \varepsilon_q^e + \varepsilon_q^p$

See “Appendix A” in workshop file package for full definitions/derivations of 3D generalization

Integration: why ?

$$\Delta\sigma_{1,2,3} = f(\sigma_{1,2,3}\,,\,e)\ \delta\varepsilon_q{}^p$$

$$\Rightarrow\quad \sigma = \int_{path} f(\sigma, e)\ \delta\varepsilon_q^p$$

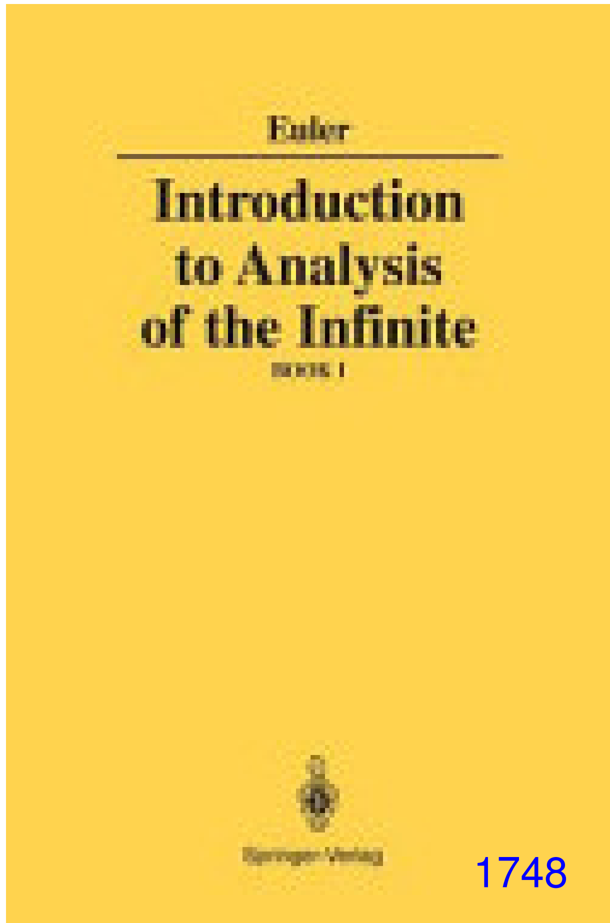
Integration: how

- No closed form solutions (mostly)…. NUMERICS

- General situations
  - stress, strain, state varies across loaded body
  - finite element or finite differences

- Laboratory tests
  - “element” tests with soil at “uniform” stress and strain state
  - stress or strain paths controlled by test arrangement (txl : $\Delta q = 3\ \Delta p$)
  - allows direct numerical integration of plasticity model

Integration: method



Euler's method